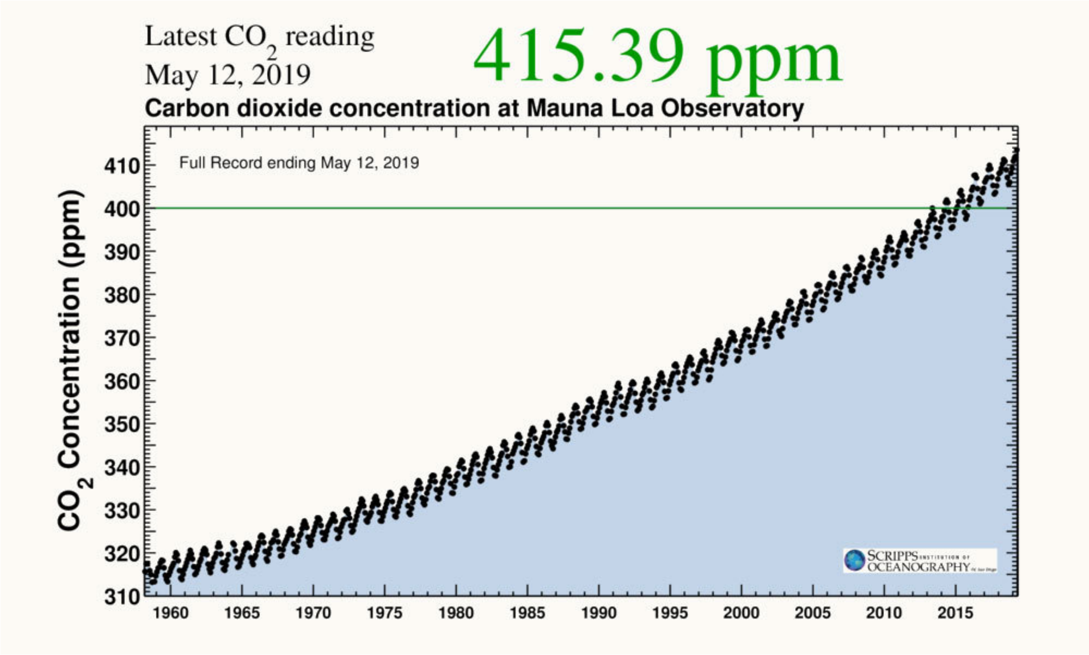
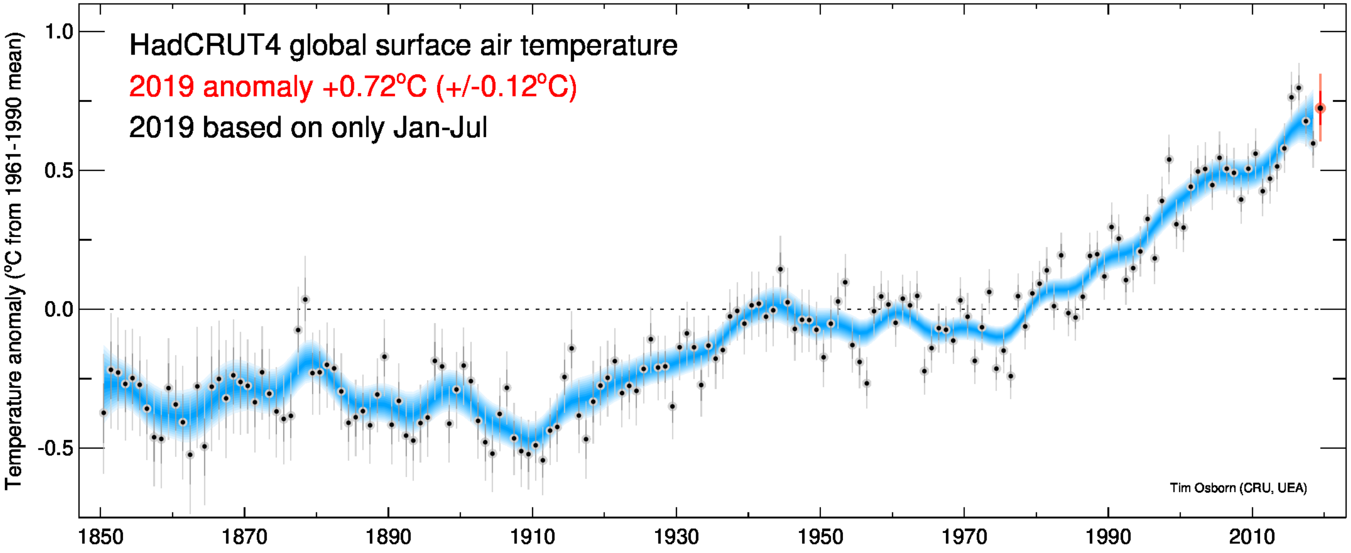
# Applications of linear response theory in climate change research

Claudia Wieners and Henk A. Dijkstra, Centre for Complex Systems Studies,
Department of Physics, Utrecht University, The Netherlands

work with Koen Helwegen (UU), M. Aengenheyster (UU, now Oxford Univ.),
F. van der Ploeg (Oxford Univ.), J. Frank (UU) and Qingyi Feng (UU)

# Climate Change: observations





# Main questions?

How warm is it going to be in 2100?

Climate Sensitivity

Are the changes going to be ‘smooth’ or ‘bumpy’?

Tipping Points

When is it too late to act to prevent dangerous climate change through emission reductions?

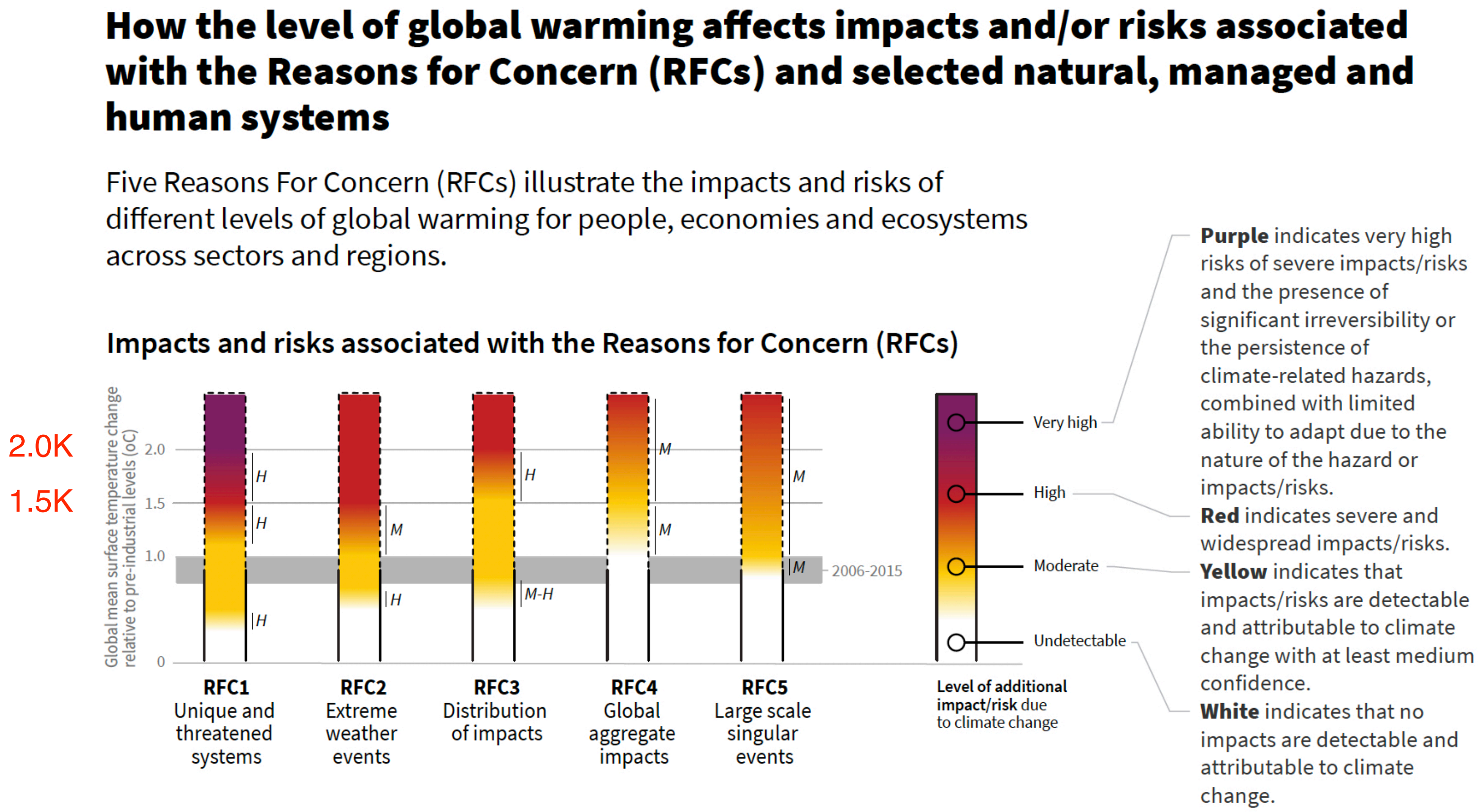
Safe Carbon Budget, Point of No Return

Are there any alternatives to avoid dangerous climate change?

Negative emissions, Geoengineering

# Dangerous Climate Change?

IPCC SR1.5



## How the level of global warming affects impacts and/or risks associated with the Reasons for Concern (RFCs) and selected natural, managed and human systems

Five Reasons For Concern (RFCs) illustrate the impacts and risks of different levels of global warming for people, economies and ecosystems across sectors and regions.

### Impacts and risks associated with the Reasons for Concern (RFCs)

**Purple** indicates very high risks of severe impacts/risks and the presence of significant irreversibility or the persistence of climate-related hazards, combined with limited ability to adapt due to the nature of the hazard or impacts/risks.

**Red** indicates severe and widespread impacts/risks.

**Yellow** indicates that impacts/risks are detectable and attributable to climate change with at least medium confidence.

**White** indicates that no impacts are detectable and attributable to climate change.

IPCC, Special Report, October 2018

# Tools

- Coordinated multi-model experiments with GCMs
- "CMIP5 provides a framework for coordinated climate change experimentation" (Taylor, Stouffer, and Meehl, 2012)
- More than 20 modeling groups
- More than 50 models
- Different design ideas, formulations, parameterizations
- Ensemble of "climate realizations"

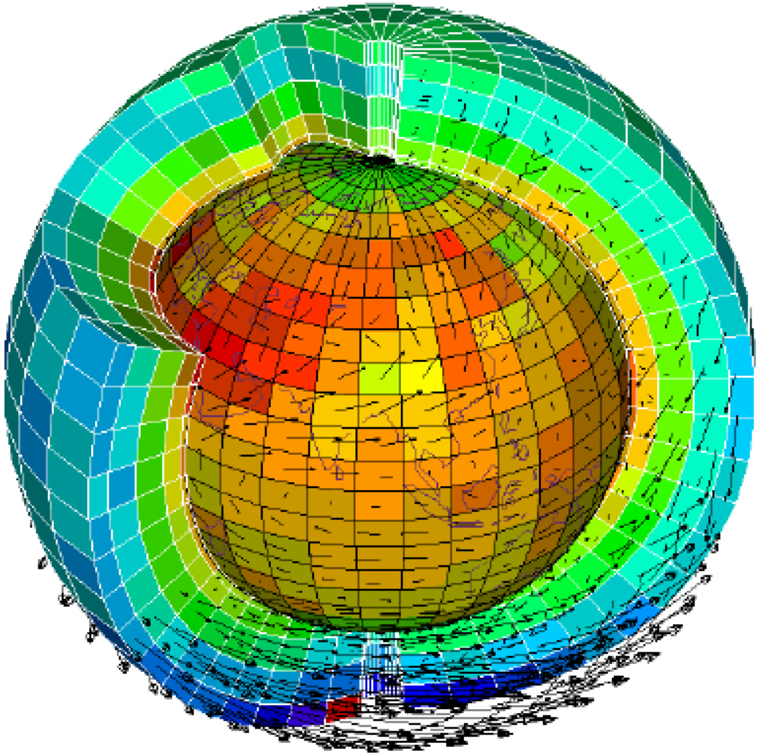
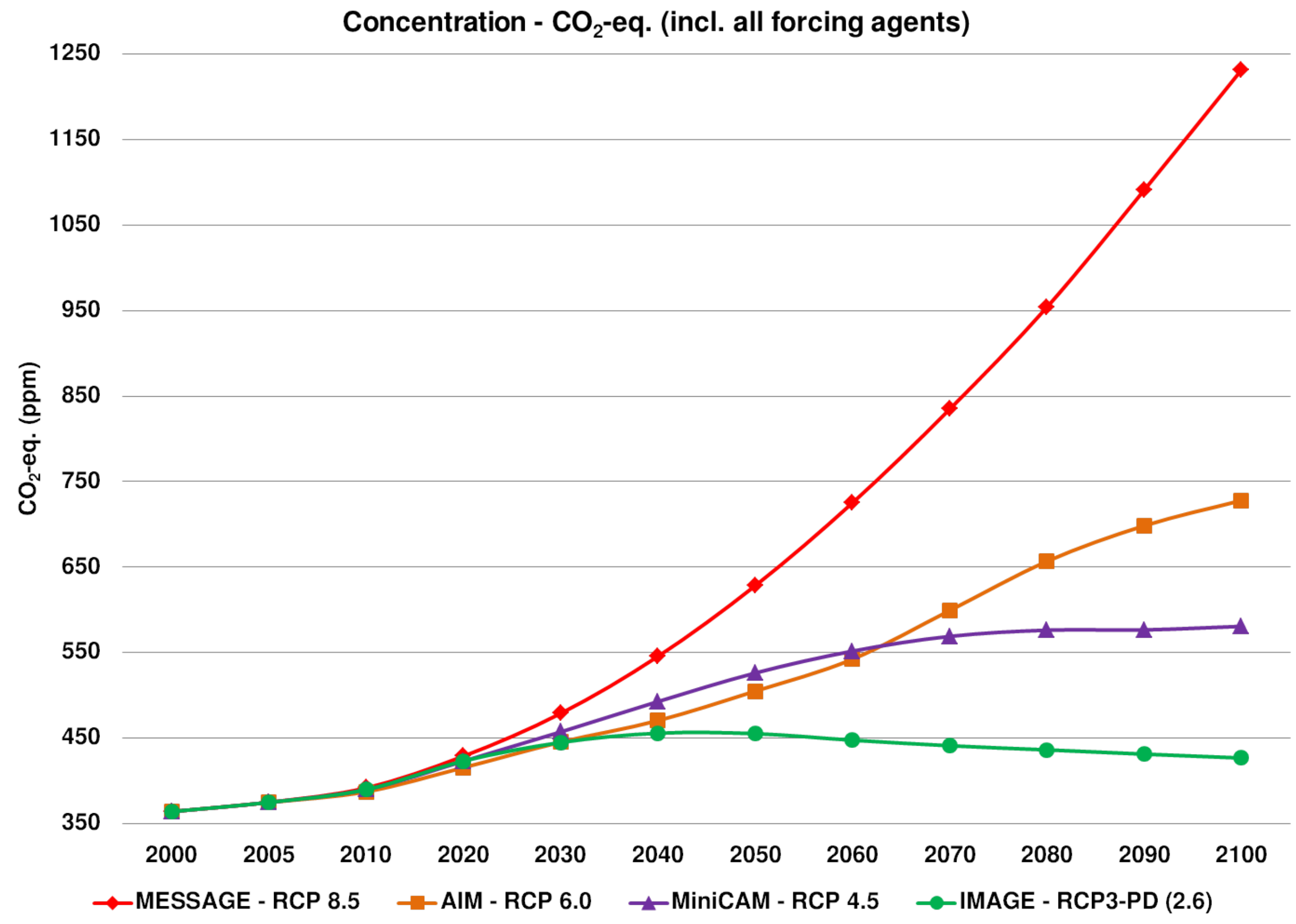


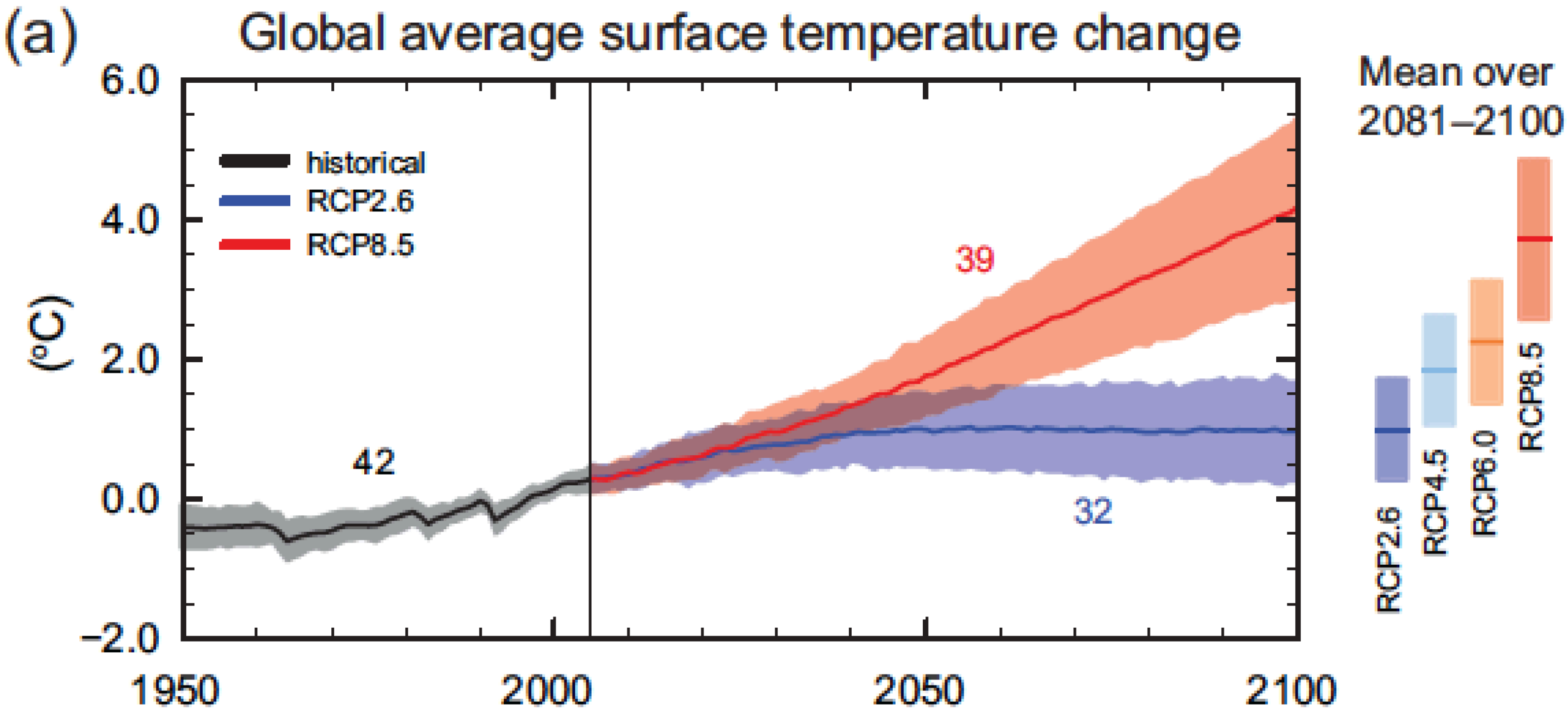
Figure: Global Climate Model (Thual, 2013)

# Future Climate Change:
# Representative concentration pathways



IPCC, Chapter 11, 2013

# Projection results



IPCC, 2013

# How to avoid dangerous climate change?

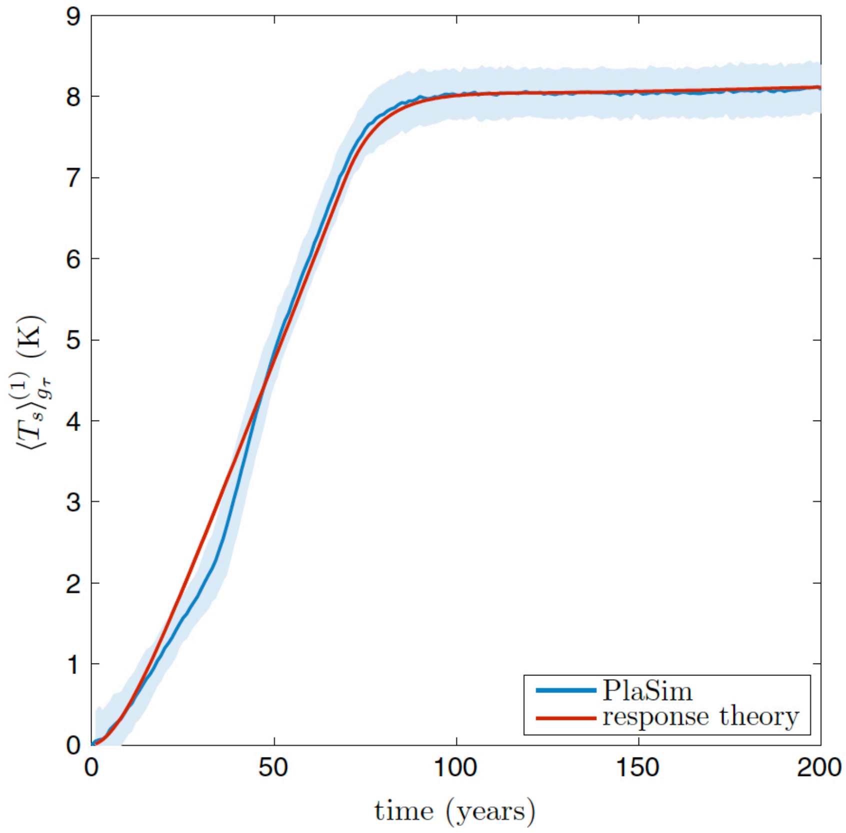
- Determining  what happens under different scenario’s

- Evaluate effects of action choices

- Include uncertainty

Need for an efficient (stochastic) climate (incl. carbon cycle) model !

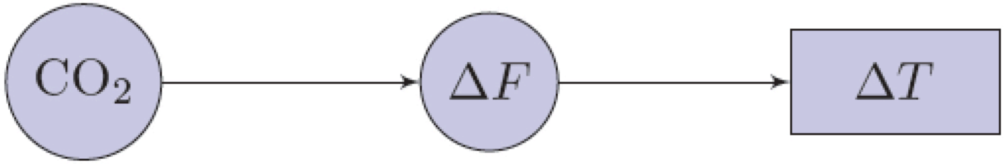
# Linear Response Theory (LRT)



Ruelle (1998)
Ragone et al, (2016)
Lucarini et al. (2017)

Using LRT  one can determine the response to any forcing!

# Procedure



- Perturbation theory

$$\Delta T_{\Delta F}(t) = \Delta T_0 + \sum_{n=1}^{\infty} \Delta T_{\Delta F}^{(n)}(t) \tag{1}$$

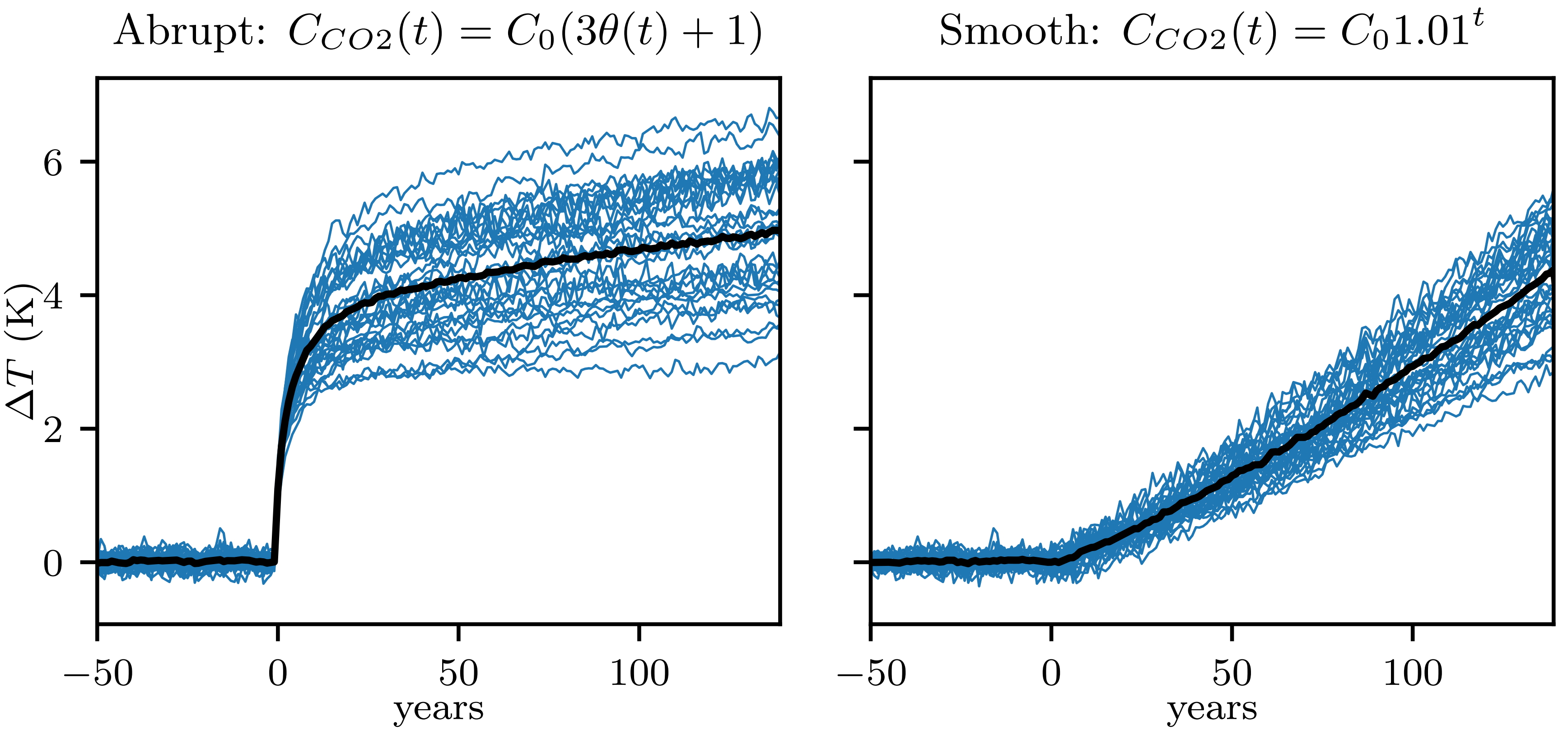
- Linear Response Theory: stop series at $n = 1$

$$\Delta T_{\Delta F}^{(1)}(t) = \int_0^t G_T(t') \Delta F(t - t') \; dt' \tag{2}$$

- Take a forcing-response pair $\Delta F_{abrupt}(t) = A\theta(t), \Delta T_{abrupt}(t)$

$$G_T(t) = \frac{1}{A} \frac{d}{dt} \Delta T_{abrupt} \tag{3}$$

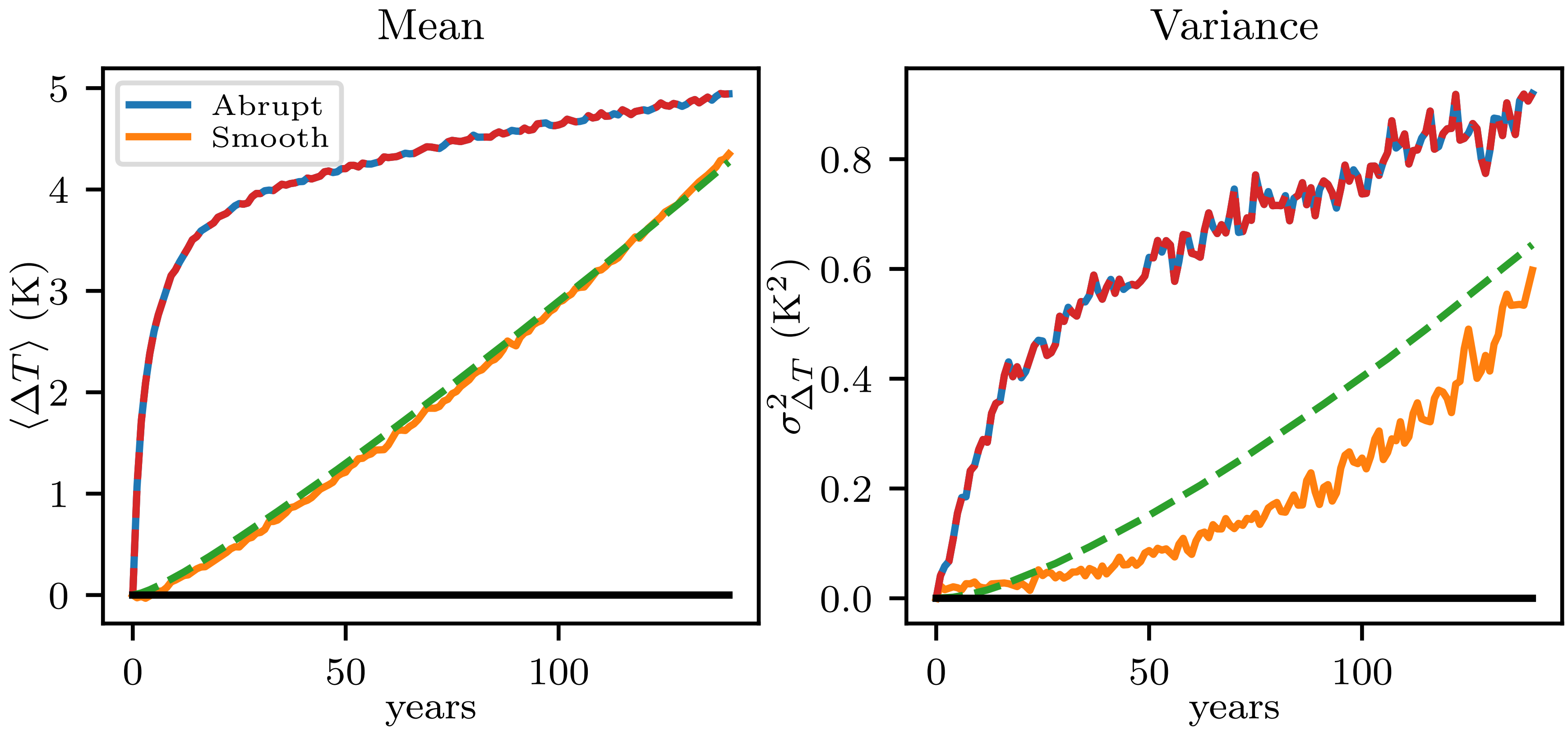
# CMIP5 simulations



Abrupt forcing

Smooth forcing

# Results Linear Response Theory



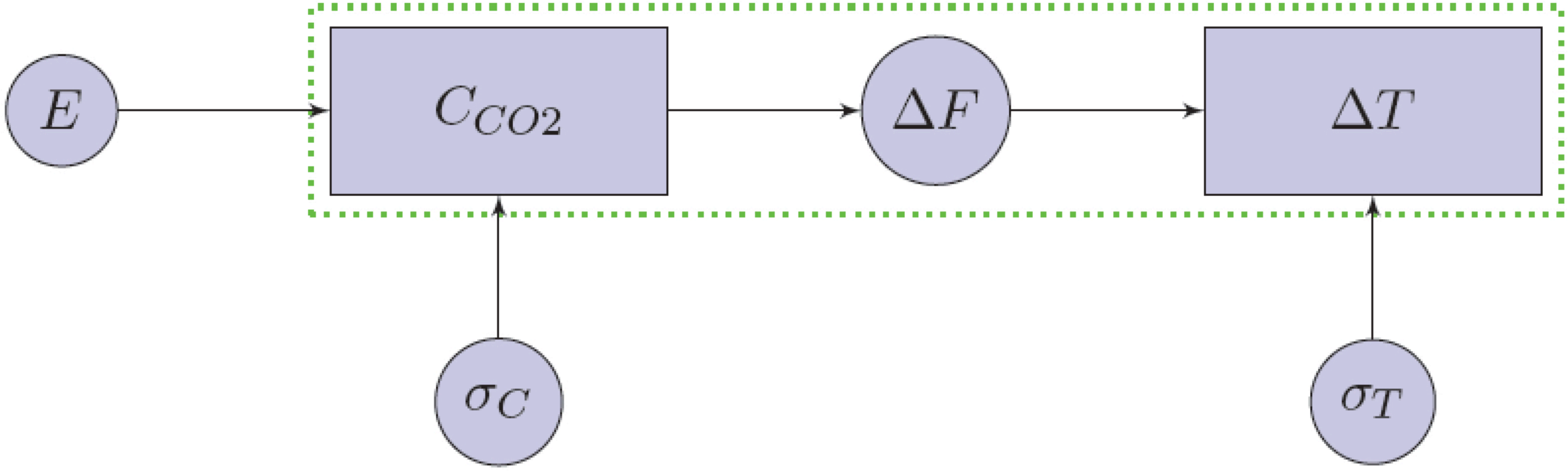
solid: CMIP5

dashed: LRT

Aengenheyster et al. (2018)

# From LRT to Stochastic State Space Model

- Up to now: determine $\Delta T$ from $CO_2$ concentrations
- Climate is forced by *fluxes*
- Relate $CO_2$ <u>emissions</u> to concentrations
- Risk, uncertainty: introduce stochasticity



# Coupling a Carbon Model

- Carbon Model (Joos et al., 2013):

$$G_{CO2}(t) = a_0 + \sum_{i=1}^{3} a_i e^{-\frac{t}{\tau_i}} \tag{4}$$

- Full Reponse Function Model

$$C_{CO2}(t) = C_{CO2,0} + \int_0^t G_{CO2}(\tau)\ E_{CO2}(t-\tau)\ d\tau \tag{5}$$

$$\Delta F_{CO2} = A\ \alpha_{CO2} \ln(C/C_0) \tag{6}$$

$$\Delta T(t) = \int_0^t G_T(\tau) \Delta F_{CO2}(t-\tau) d\tau \tag{7}$$

- We also find:

$$G_T(t) = \sum_{i=0}^{2} b_i e^{-t/\tau_{bi}} \tag{8}$$

# Stochastic State Space Model

## Carbon

$$dC_P = a_0 E dt$$

$$dC_1 = \left( a_1 E - \frac{1}{\tau_1} C_1 \right) dt$$

$$dC_2 = \left( a_2 E - \frac{1}{\tau_2} C_2 \right) dt + \sigma_{C2}\ dW_t$$

$$dC_3 = \left( a_3 E - \frac{1}{\tau_3} C_3 \right) dt$$

$$C_{CO2} = C_P + \sum_{i=1}^{3} C_i$$

## Temperature

$$\Delta F = A\ \alpha \ln(C_{CO2}/C_0)$$

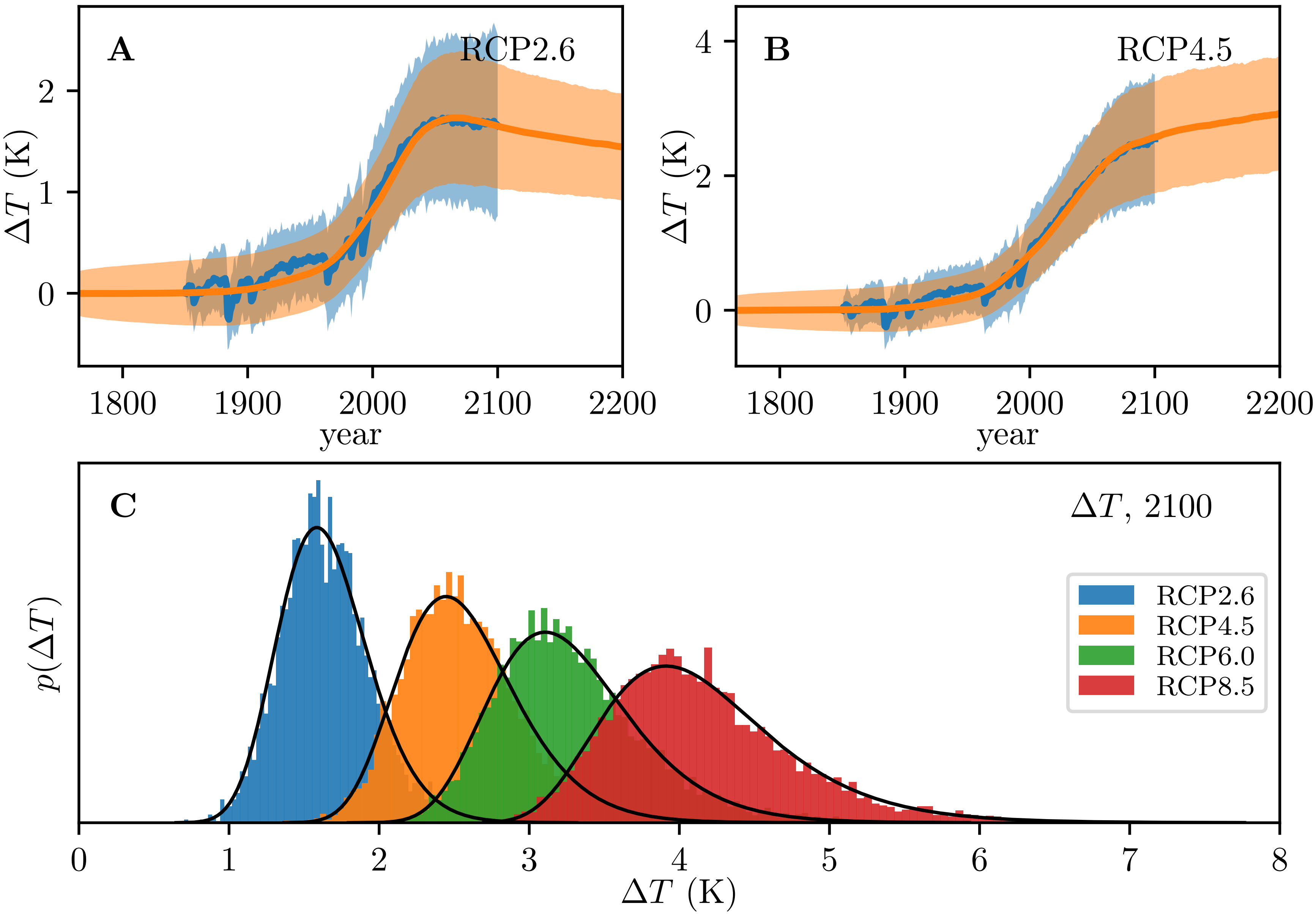
$$d\Delta T_0 = \left( b_0 \Delta F - \frac{1}{\tau_{b0}} \Delta T_0 \right) dt + \sigma_{T0}\ dW_t$$

$$d\Delta T_1 = \left( b_1 \Delta F - \frac{1}{\tau_{b1}} \Delta T_1 \right) dt$$

$$d\Delta T_2 = \left( b_2 \Delta F - \frac{1}{\tau_{b2}} \Delta T_2 \right) dt + \sigma_{T2} \Delta T_2\ dW_t$$

$$\Delta T = \sum_{i=0}^{2} \Delta T_i$$

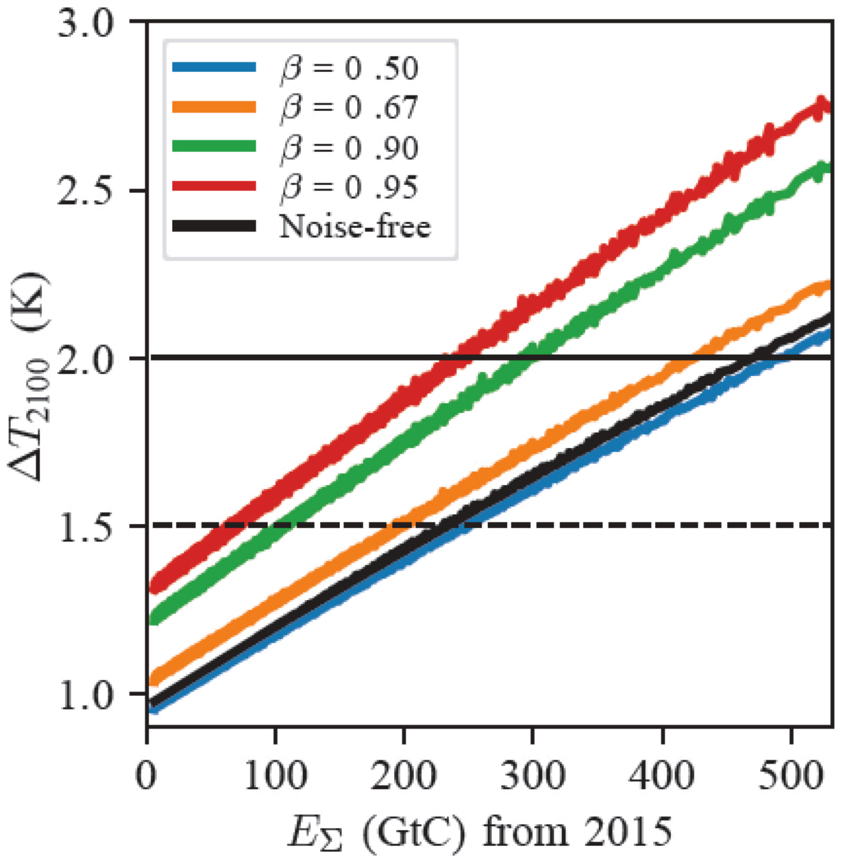
# Results: Probability density functions



# Save Carbon Budget

Maximum Cumulative Emissions that reach a certain warming target

$$p(\Delta T \leq T_{max}) = \beta$$



$$T_{max} = \Delta T_{2100}$$

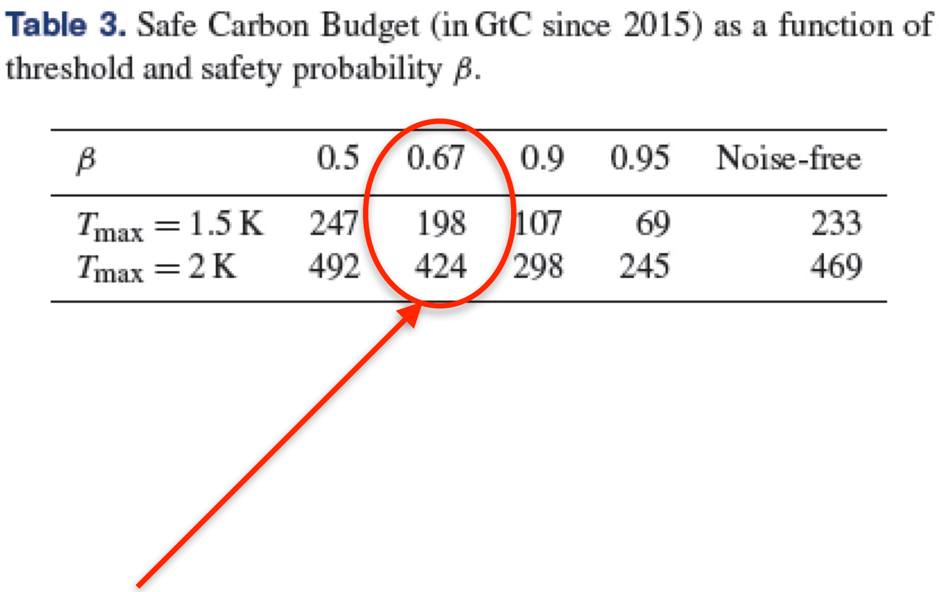


**Table 3.** Safe Carbon Budget (in GtC since 2015) as a function of threshold and safety probability $\beta$.

| $\beta$ | 0.5 | 0.67 | 0.9 | 0.95 | Noise-free |
|---|---|---|---|---|---|
| $T_{max} = 1.5$ K | 247 | 198 | 107 | 69 | 233 |
| $T_{max} = 2$ K | 492 | 424 | 298 | 245 | 469 |

IPCC-AR5: 377 - 517 GtC to likely stay below 2 K

Millar et al. (2017): 200 GtC to likely stay below 1.5 K

# Economy & Transition pathways

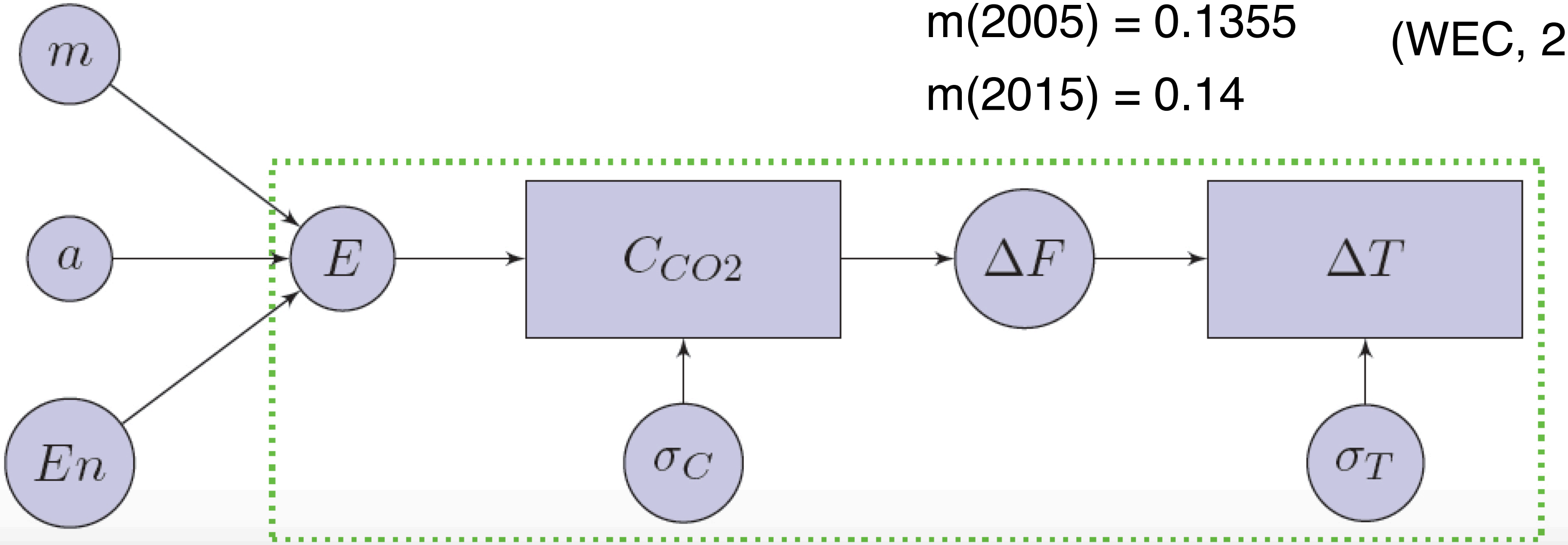
$$Y = Y_0 e^{gt}$$
$$En = \gamma_0 e^{-r_\gamma t} Y$$
$$E = (1-a)(1-m)En$$

1. Extreme Mitigation (EM): From time $t_a$ on, we set $m = 1$, i.e. $E = 0$.
2. Fast Mitigation (FM): From time $t_a$ on, both $a, m$ increase by 0.05 per year.
3. Ambitious Mitigation (AM): As FM, but the increase is 0.02 per year.

m(2005) = 0.1355

m(2015) = 0.14

(WEC, 2016)



# Point of No Return (PONR)

- Use economical assumptions to determine emissions $\Rightarrow$ baseline ‘business-as-usual’ scenario
- Control emissions by mitigation $m(t)$ and abatement $a(t)$ $\Rightarrow$ actions on climate change modify ‘business-as-usual’ scenario

### Definition (Point of No Return)

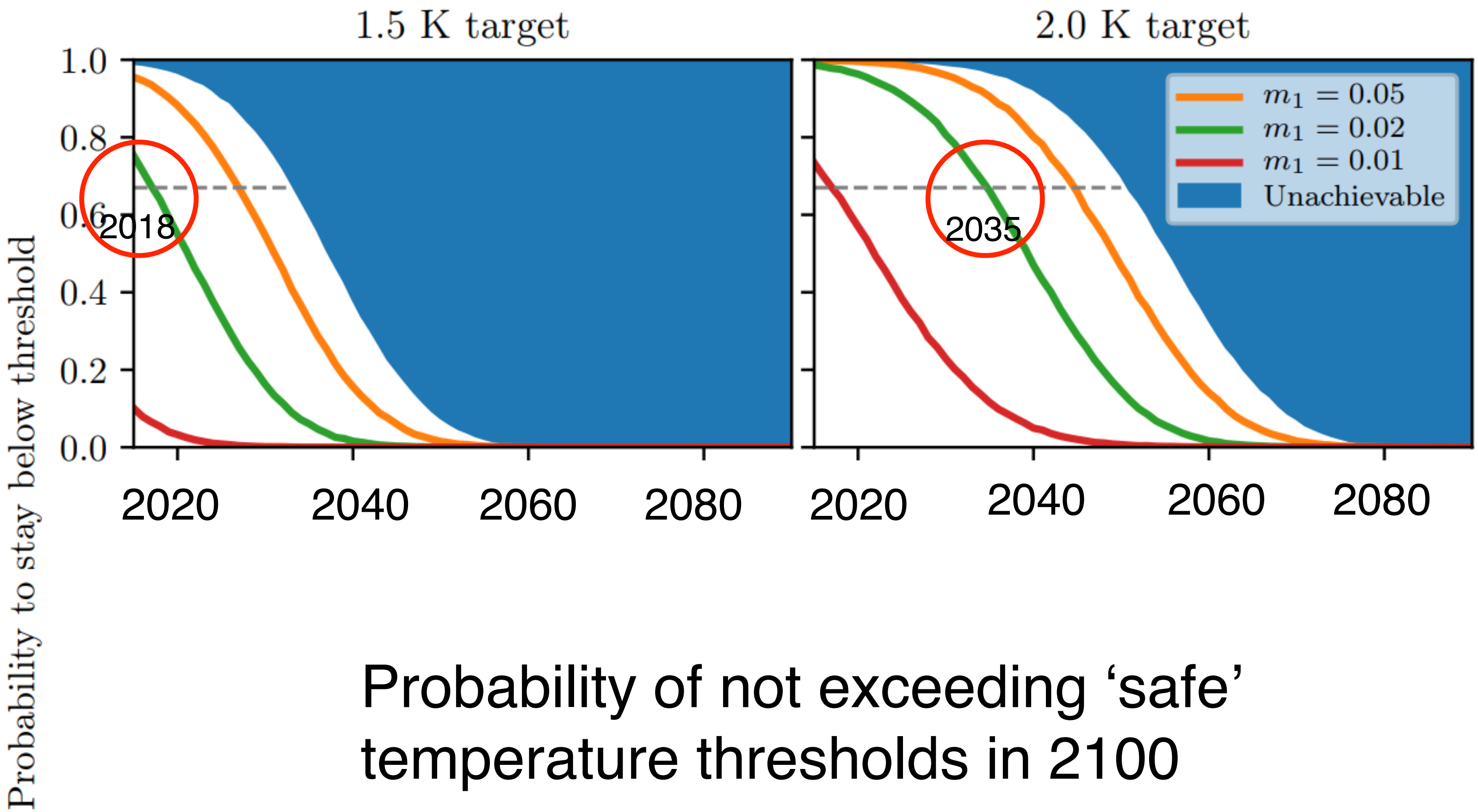
*The Point of No Return (PONR) is the time $t_P$ from which on no* ***allowed*** $[a(t), m(t)]$ *such that* $0 \leq a(t), m(t) \leq 1, t_P \leq t \leq t_f$ *can be chosen to fulfill*

$$p(\Delta T(t_f) \leq T_{max}) \geq \beta$$

Save Carbon Budget:
We cannot reach target X by emitting more than Y

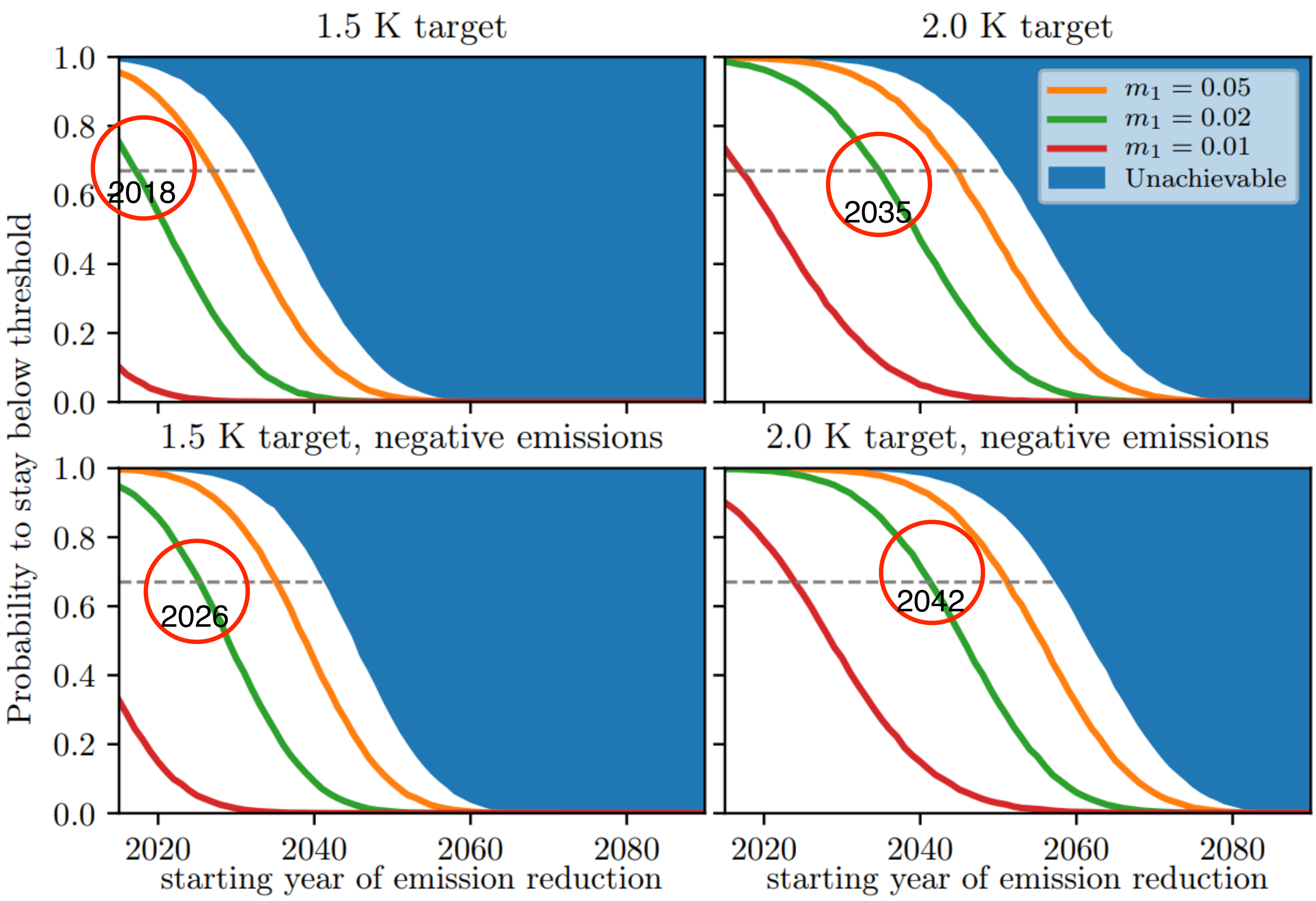
Point of No Return:
We cannot reach target X by starting reduction after year T

# PONR: results



Probability of not exceeding ‘safe’ temperature thresholds in 2100

# Effect of Negative Emissions



Probability of not exceeding ‘safe’
temperature thresholds in 2100

# PONR: results

1. Extreme Mitigation (EM): At time $t_a$, we set $m = 1$, so $E = 0$ from then onward.
2. Fast Mitigation (FM): From time $t_a$ onwards, both $a, m$ increase by 0.05 per year.
3. Ambitious Mitigation (AM): As FM, but the increase is 0.02 per year.

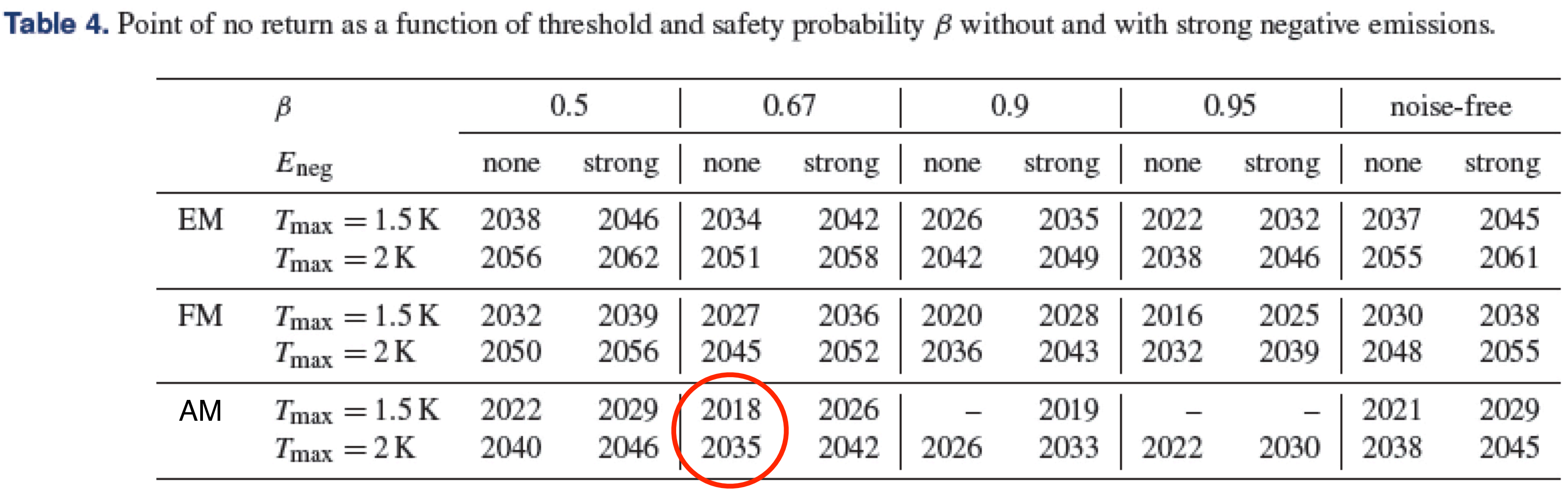


**Table 4.** Point of no return as a function of threshold and safety probability $\beta$ without and with strong negative emissions.

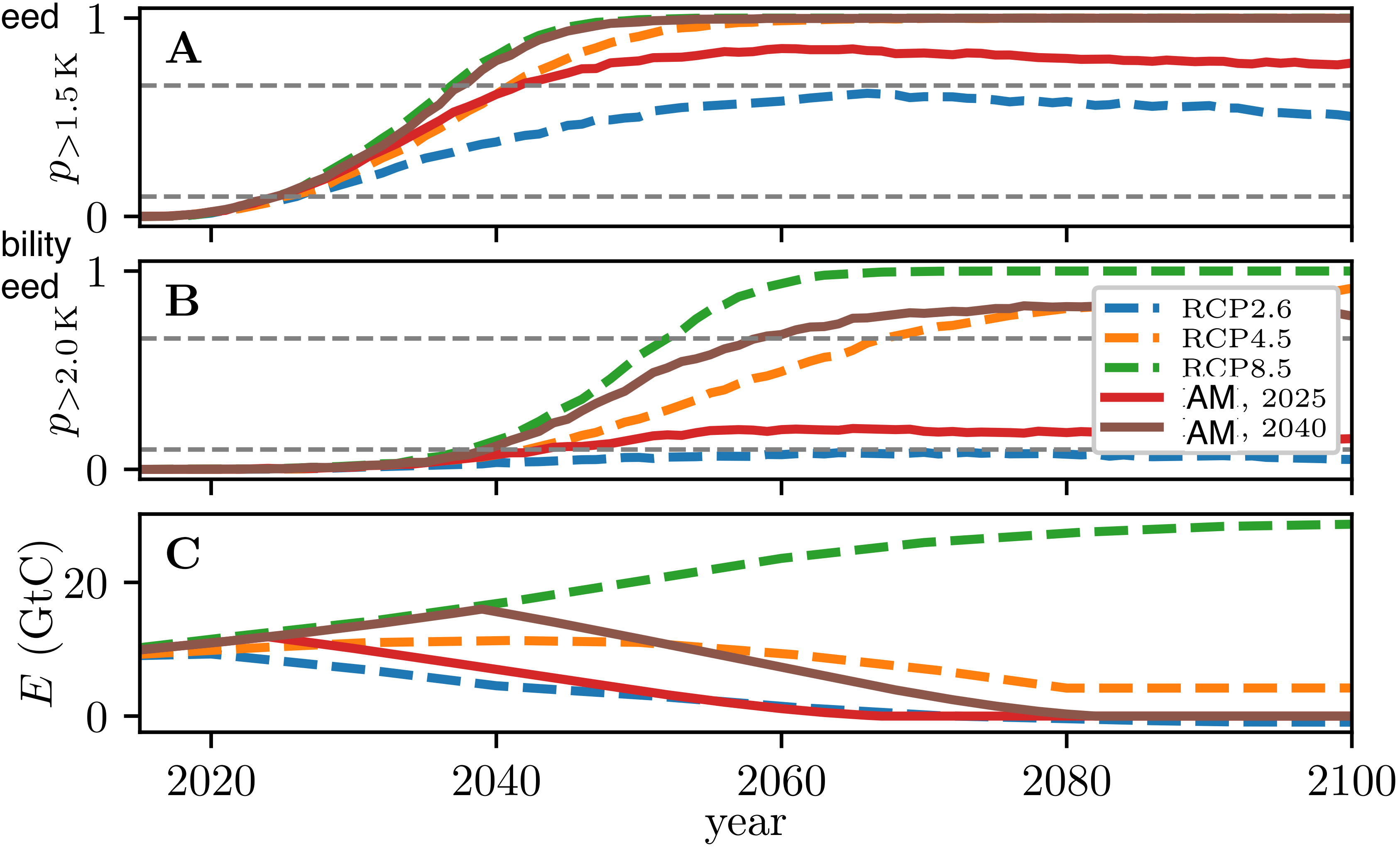
| | $\beta$ | 0.5 | | 0.67 | | 0.9 | | 0.95 | | noise-free | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $E_{\text{neg}}$ | none | strong | none | strong | none | strong | none | strong | none | strong |
| EM | $T_{\max} = 1.5\,\text{K}$ | 2038 | 2046 | 2034 | 2042 | 2026 | 2035 | 2022 | 2032 | 2037 | 2045 |
| | $T_{\max} = 2\,\text{K}$ | 2056 | 2062 | 2051 | 2058 | 2042 | 2049 | 2038 | 2046 | 2055 | 2061 |
| FM | $T_{\max} = 1.5\,\text{K}$ | 2032 | 2039 | 2027 | 2036 | 2020 | 2028 | 2016 | 2025 | 2030 | 2038 |
| | $T_{\max} = 2\,\text{K}$ | 2050 | 2056 | 2045 | 2052 | 2036 | 2043 | 2032 | 2039 | 2048 | 2055 |
| AM | $T_{\max} = 1.5\,\text{K}$ | 2022 | 2029 | 2018 | 2026 | – | 2019 | – | – | 2021 | 2029 |
| | $T_{\max} = 2\,\text{K}$ | 2040 | 2046 | 2035 | 2042 | 2026 | 2033 | 2022 | 2030 | 2038 | 2045 |

The PNR for 1.5 K has been passed!

# Scenarios

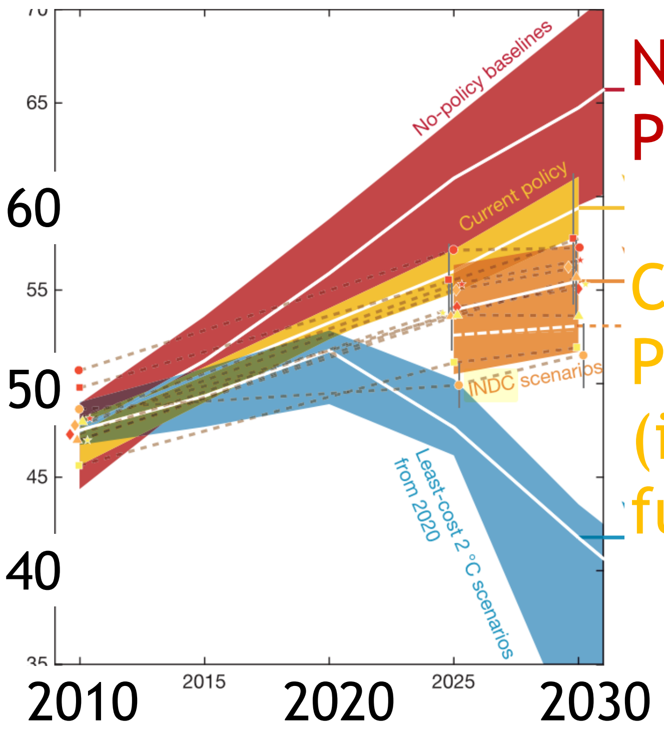
Probability
to exceed
1.5 K

Probability
to exceed
2 K



# Current Climate Policy

annual GHG emissions (Gt(CO2 eq) / year)



NO POLICY

CURRENT PLEDGES (if fullfilled)

NEEDED TO REACH EVEN 2°C

**Even if all states keep their intended contributions, we are NOT on the right path to reach the Paris agreement!**

Rogelj et al. 2016

# Summary

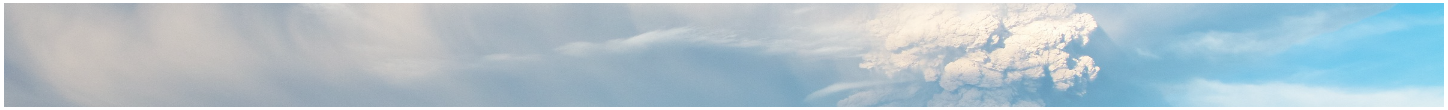
- Shown value of Linear Response Theory in the CMIP5 context

- Build an stochastic state space model to adequately determine th climate response to emissions.

- Easy to communicate metrics:
  - SCB: “We cannot reach target X when emitting more than
  - PONR: “We cannot reach target X when starting after year PONR”

PONR with realistic action pathways is close (2035 for 67%) for the 2K target and already passed for the 1.5K target.

# Further reading



**M. Aengenheyster, Q. Feng, F. van der Ploeg and H. A. Dijkstra. The point of no return for climate action: effects of climate uncertainty and risk tolerance Earth System Dynamics, 9, 1085-1095, (2018)**

**K. Helwegen, C. Wieners, J. Frank and H. A. Dijkstra. Complementing $CO_2$ emission reduction by solar radiation management might strongly enhance future welfare, Earth System Dynamics, 10, 453-472, (2019).**