Extended Dynamical Systems on Discrete Lattices and Application to Physics and Biophysics for Energy Localisation and Transport

Many classical Hamiltonian dynamical systems on finite or extended discrete lattices may sustain Discrete Breathers (DB) which are exact spatially localized and time periodic solutions. DBs are intrinsically nonlinear objects which may spontaneously manifest under some specific conditions by focusing energy (or charge) over long times and generate unusual physical phenomena. They may be also involved in the transport of energy (or charge). The course will mix mathematics and physics. We shall present what is known about DBs, exact and emprical results and also discuss applications, open problems and new possible directions in theoretical physics.

After an introduction where we shall present the physical motivation to study DBs, the beginning of the course will focus on the mathematical aspect of the theory of DBs (and multibreathers) including exact existence proofs, numerical results and conjectures. Some of the existence proofs of DBs, which hold for different models and hypothesis, will be described in detail. We shall also discuss the existence of DBs (and multibreathers) in random lattices and their connection with Anderson localisation. Next, we shall investigate the linear stability of DBs, the Krein theory in term of a *band analysis* and DBs bifurcations. Interaction of DBs with a linear phonon will be discussed at lowest order (linear phonon scattering, inelastic interactions). Mobile DBs may be observed in some circonstances.

The second part of the course will be devoted to physical applications where energy (or charge) focusing is essential and which are (partially) supported by the previous mathematical results. We shall first briefly review some physical problems where the role of DBs have already been suggested to be important (Davydov soliton, DNA, ...). We shall next describe in detail the new concept of Targeted Transfer applicable for the selective transportation of DBs, polarons or other quantum excitations. We shall propose new applications for understanding chemical reactions which do not obey the standard activation theory (Kramers). We shall suggest the existence of *chemical expressways* and illustrate the concept by two examples: a toy model for ultrafast and selective chemical dissociation and a model for ultrafast Electron Transfer in the Photosynthetic Reaction Center. We shall conclude on new directions of research especially for biophysics.