3 COMMENTS ON "LINEAR RESPONSE FOR SMOOTH DEFORMATIONS OF GENERIC NONUNIFORMLY HYPERBOLIC UNIMODAL MAPS"

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1) In the Beginning of §5.2, it is claimed incorrectly that 1 is always the only eigenvalue of the transfer operator on the unit circle. Since we did not assume mixing, there could be in fact finitely many other simple eigenvalues of modulus one in general (they are roots of unity — see the proof of Proposition 3.6 in Appendix B of [1] and the reference [3] to Karlin there). So, when constructing the contour integrals (112) (Step 1 in §6), we should avoid not only a neighbourhood of the disc of radius θ_t there (see also (73) in [1]), but also neighbourhoods of these other eigenvalues of modulus 1 (see the circle γ in the proofs of Propositions 4.1 and 4.2 of [1], and note that Proposition 4.1 should be proved before Proposition 4.2.). Note also that exponential decay of correlations is not needed (up to replacing $\widehat{\mathcal{L}}^n$ by $k^{-1}\sum_{n=0}^{k-1}\widehat{\mathcal{L}}^n$ in the proof of the last claim of Proposition 4.11, see [1, 142]). Finally, since the renormalisation period is constant, note that $\theta_t < 1$ is uniform, by [1, Propositions 4.1–4.2].

2) We cannot apply the exactness argument from [2, Corollary 2] to show that 1 is a simple eigenvalue for a nonnegative eigenvector of the transfer operator (contrarily to what is stated in the proof of Proposition 4.11), because the transfer operator $\widehat{\mathcal{L}}_t$ is associated to a probabilistic and not a deterministic tower map. However, we may apply classical results on positive operators [3] (details are given in Appendix B of [1]).

3) To prove (115) one should apply the fundamental theorem of calculus instead of the mean value theorem.

References

[1] V. Baladi, M. Benedicks, and D. Schnellmann. Whitney-Hölder continuity of the SRB measure for transversal families of smooth unimodal maps. *arXiv:1302.3191*.

[2] Viviane Baladi and Marcelo Viana. Strong stochastic stability and rate of mixing for unimodal maps. *Ann. Sci. École Norm. Sup. (4)*, 29(4):483–517, 1996.

[3] Samuel Karlin. Positive operators. *J. Math. Mech.*, 8:907–937, 1959.

Date: February 18, 2013.