CORRIGENDUM (DECEMBER 4, 2002)
ALMOST SURE RATES OF MIXING FOR I.I.D. UNIMODAL MAPS

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The definition of the return time on p. 117 and the beginning of the proof of Proposition 8.3 on pp. 118–120 are not correct. Indeed the large image property claimed in the middle of p. 120 is not guaranteed by the construction, and the interplay between the hyperbolic-time partition in §7.3 and Lemma 7.10 must be monitored more carefully. The amended version is given below. None of the statements are affected by this.

(Note also that the right-hand-side of the bound in Sublemma 7.2(3) should be $\max(e^{-2\eta k}, \epsilon^\eta)\min(e^{2k}, \epsilon^{-1})$. As a consequence, Lemma 7.10 is only true for $k \le (1/2)\log(1/\epsilon)$. This does not cause problems in Lemma 8.1: if $1-3\eta >= 1/2$, continue iterating, obtaining $1-5\eta$, $1-9\eta$ etc until $1-\ell\eta < 1/2$.)

V.B. thanks S. Gouëzel for pointing out the mistake and for very helpful discussions.

Corrections in §8

The definition of escape times on pp. 116–117 and Cor. 8.2 on p. 117 are not neeeded. Replace the text after the proof of Lemma 8.1 (p. 116) and before "Estimate (8.3) for $\widehat R_\omega$" (p. 119) by:

Proposition 8.3 (Partition $\Lambda_i(\omega)$ and return times R_ω). *There exist $\Omega_1 \subset \Omega_0$ of full measure, and for all $\omega \in \Omega_1$, a countable (modulo zero measure sets) partition $\mathcal{Z}_\omega$ of Λ, into subintervals $\{\Lambda_i = \Lambda_i(\omega) \mid i \in \mathbb{Z}_+\}$ and a return-time function R_ω, defined almost everywhere on Λ, and satisfying Axioms (A.I)–(A.IV) from Section 3.2.*

Additionally, there are a random variable $n_1(\omega)$ and constants $C(\epsilon) \ge 1$, $C_1(\epsilon) > C_2(\epsilon) > 1$ so that for all $\ell \ge n_1(\omega)$

$$\mathrm{Leb}(\{x \in \Lambda \mid R_\omega(x) > \ell\}) < C(\epsilon) e^{-(\ell^{\frac14}/C_1(\epsilon))},$$
$$P(\{\omega \in \Omega_1 \mid n_1(\omega) > \ell\}) < C(\epsilon) e^{-(\ell^{\frac14}/C_2(\epsilon))}.$$

We may replace the right-hand-sides in both inequalities by $C(\epsilon)e^{-\ell^u}$ for $0 < u < 1/4$.

Proof of Proposition 8.3. We first define a partition $\widehat\Lambda_i(\omega)$ and estimate return times $\widehat R_\omega(x)$ corresponding to the first moment when one of the $\Lambda_\pm$ is exactly covered (with expansion and distorsion control).

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Fix $\omega \in \Omega$. Our starting point is the countable decomposition from §7.3:

$$\Lambda_+ \cup \Lambda_- = \cup_{m=p_0}^{\infty} \bigcup_{J \in \mathcal{R}_m(\omega)} J\,,$$

where we have taken $p_0 \geq C \log(1/\epsilon)$ (the bound on $t(J)$ from Lemma 8.1) and we have introduced the intervals $\Lambda_\pm$ and $\widetilde{\Lambda}_\pm \setminus \Lambda_\pm$ in the initial partition $\mathcal{Q}$. For (almost) each $x \in \Lambda_+ \cup \Lambda_-$, there is $J \in \mathcal{R}_m(\omega)$ containing x. We shall further partition it (if needed). By construction, almost every initial x will be contained in an interval $\widehat{J}$ of the subpartition, and Lemma 7.9 (2,3) will hold (injectively) for $\widehat{J}$ and an iterate $\hat{m}(\widehat{J}) < \infty$. Additionally, the Markov property $f^{\hat{m}}(\widehat{J}) = \Lambda_+$ or $f^{\hat{m}}(\widehat{J}) = \Lambda_-$ will hold. The partition $\widehat{\Lambda}_i$ will be the partition into intervals $\widehat{J}$, and the return time on $\widehat{J}$ will be $\widehat{R}_\omega \equiv \hat{m}(\widehat{J})$.

To control the return-time asymptotics, we shall introduce a sequence of *stopping times* $\widehat{T}_i = \widehat{T}_{\omega,i} : \Lambda_+ \cup \Lambda_- \to \mathbb{N} \cup \{\infty\}$ with...
(no more changes on p.118 until after (8.5)*, except that Lemma 7.8 should be mentioned instead of Corollary 8.2.)*

Let us now define the stopping times, using again the notation from Lemma 8.1. We say that $\widehat{T}_{\omega,1}$ is defined at $x \in \Lambda_+ \cup \Lambda_-$ if there is $J_1 \in \mathcal{R}_{m_1}(\omega)$ with $x \in J_1$ (hence, $f^{m_1+t(J_1)}(J_1)$ contains $f^{m_1+t(J_1)}(x)$ and covers $\widetilde{\Lambda}_+$ or $\widetilde{\Lambda}_-$). We then set

$$\widehat{J}_1 = (f^{m_1}_\omega)^{-1} \circ (f^{t(J_1)}_{\sigma^{m_1}\omega}|_{\tilde{J}_1})^{-1}(\Lambda_+) \text{ or } \Lambda_-$$

$$\text{and } \widehat{T}_{\omega,1}(x) = \begin{cases} m_1 + t(J_1) = \widehat{R}_\omega(x) & \text{if } x \in \widehat{J}_1, \\ m_1 & \text{otherwise.} \end{cases}$$

The leftover interval(s) $J_1 \setminus \widehat{J}_1$, correspond to points which have reached a stopping time but not returned yet. They will be further partitioned. For this, let $\Theta_{\omega,1} = \{x \in \Lambda_+ \cup \Lambda_- \mid \widehat{T}_{\omega,1}(x) \text{ is defined}\}$. For $x \in \Theta_{\omega,1} \setminus \{\widehat{R}_\omega(x) = \widehat{T}_{\omega,1}(x)\}$, we say that $\widehat{T}_{\omega,2}$ is defined at x if there are $m_2 > p_0$ and $J_2 \in \mathcal{R}_{m_2}(\sigma^{\widehat{T}_{\omega,1}(x)}\omega, L_1(x))$ with $f^{\widehat{T}_{\omega,1}(x)}_\omega(x) \in J_2$. Here, the intervals in $\mathcal{R}_{m_2}(\sigma^{\widehat{T}_{\omega,1}(x)}\omega, L_1)$ are obtained by applying the partitioning rules in §7.3 to the interval in $L_1 = L_1(x) = f^{m_1}_\omega(J_1 \setminus \widehat{J}_1)$ containing $f^{m_1}_\omega(x)$. We describe this new mechanism in detail next, noting first that it may produce a different partition than the intersection of L_1 with the partition described in §7.3, but that Lemma 8.1 will hold for the modified partition (because a modified version of Lemma 7.9 will hold).

More precisely, when applying the mechanism in §7.3 to a subinterval $L' \subset I$ (for example $L' = L_1(x)$) (P.I) should be replaced by:
($\widehat{\text{P}}$.I): $H^*_m(\omega') \cap L' \subset \cup_{j=p_0}^m \bigcup_{J \in \mathcal{R}_j(\omega', L')} J$, and $J \cap H^*_m(\omega') \neq \emptyset$ for all $J \in \mathcal{R}_m(\omega', L')$ and $m \geq p_0$.
Writing $\mathcal{S}_m(\sigma^{m_1}\omega, L_1)$ for $L_1 \setminus \cup_{j=p_0}^m \cup_{J \in \mathcal{R}_j(\sigma^{m_1}\omega, L_1)} J$, Lemma 7.8 then implies that $\mathrm{Leb}(\mathcal{S}_m(\sigma^{m_1}\omega, L_1)) \leq Ce^{-\zeta(\epsilon)m}$ for all $m \geq n_0(\sigma^{m_1}\omega)$.
(P.II) is unchanged and will hold by construction, guaranteeing the required bounded

distorsion. (P.IV) is also unchanged.
The large image property (P.III) on p. 112 (essential in the proof of Lemma 7.9(1)) must be replaced by the following modified large image property:
($\widehat{\mathrm{P}}$.III): for each $J \in \mathcal{R}_m(\omega, L)$ either (P.III) holds or there is $0 \le j \le C \log(1/\epsilon)$ so that $f^j_\omega(J)$ contains an $\hat I_{k,\ell}$. (See the proof of Lemma 7.10, pp.115–116, noting that $(-\delta, \delta)$ contains infinitely many intervals I_k. In fact, it is useful to replace the three adjacent $\hat I_{k,\ell}$s in (7.21,7.24), but not (7.25), by five such adjacent intervals.) Then, changing slightly the computations on pp.113–114 to accommodate for the k^{-2} factor in the intervals $\widehat{I}_{k,\ell}$ (note that we can assume $e^{-k} \ge \sqrt{\epsilon} e^{-r_j(\omega,x)+1}$), we get $|f^m_\omega(J)| \ge \epsilon^{1-2\eta}/(C \log(\epsilon^{2\eta-1}))$ almost as in Lemma 7.9(2), and enough for the purposes of the proof of Lemma 8.1.

Let $\widehat{J}_2 = (f_\omega^{\widehat{T}_{\omega,1}(x)+m_2})^{-1} \circ (f^{t(J_2)}_{\sigma^{m_1}\omega}|_{\tilde J_2})^{-1}(\Lambda_\pm)$. We set $\widehat{T}_{\omega,2}(x)$ to be $\widehat{R}_\omega(x) = \widehat{T}_{\omega,1}(x) + m_2 + t(J_2)$ if $f_\omega^{\widehat{T}_{\omega,1}(x)}(x) \in \widehat{J}_2$, and otherwise $\widehat{T}_{\omega,2}(x) = \widehat{T}_{\omega,1}(x) + m_2$. For general $k \ge 2$, we let $\Theta_{\omega,k} = \{x \mid \widehat{T}_{\omega,k}(x) \text{ is defined}\}$, and we define $\widehat{T}_{\omega,k+1}$ on $\Theta_{\omega,k+1} \subset \Theta_{\omega,k} \setminus \{\widehat{R}_\omega(x) = \widehat{T}_{\omega,k}(x)\}$ if there is $m_{k+1} \ge p_0$ and $J_{k+1} \in \mathcal{R}_{m_{k+1}}(\sigma^{\widehat{T}_{\omega,k}(x)}\omega, L_k)$ with $f_\omega^{\widehat{T}_{\omega,k}(x)}(x) \in J_{k+1}$. The relation (8.1) (and thus (8.2)) is an immediate consequence of the definition.

The modifications in the rest of the proof of Proposition 8.3 are:

1. Before the proof of estimate (8.3) on p.119, insert: "Note that the total distorsion may be estimated by $C(\epsilon)^i \xi^\ell$, where the return time ℓ is at least p_0 times the number i of stopping times and $\beta < \xi < 1$ (if necessary, take a larger $p_0(\epsilon)$ to ensure $C(\epsilon)^{1/p_0}\xi < 1$)."
2. In the proof of estimate (8.3) on p.119: replace "J is an interval of $\mathcal{R}_m(\sigma^{n'}\omega)$" by "$J$ is an interval of $\mathcal{R}_m(\sigma^{n'}\omega, L)$."
3. On p. 120, replace the "absolute bounds from Corollary 8.2" by "($\widehat{\mathrm{P}}$.1)" (twice).
4. On p. 120, replace "large image properties from Lemma 8.1" by "the modified large image property above."
5. Replace the notation R^*_ω by R_ω on pp. 121–22.

The rest of the paper

In Section 9, (A.IV) now comes from Proposition 8.3 (instead of Lemmas 7.9–7.10).

We take this opportunity to mention typographical mistakes in §7.3: in the first line of p.112: $H_k(\omega) \subset \cup_{J \in \mathcal{R}_k(\omega)} J$ should be replaced by $H_k(\omega) \subset \cup_{j=p_0}^k \cup_{J \in \mathcal{R}_j(\omega)} J$; in the second line $n \ge n_0(\omega)$ must be replaced by $m \ge n_0(\omega)$.

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