Correcting the proof of Theorem 3.2 in *Almost sure rates of mixing for i.i.d. unimodal maps* (V. Baladi, M. Benedicks, V. Maume-Deschamps) Ann. E.N.S. (2002)

August 27, 2010

We thank Weixiao Shen, who pointed out to us that the proof of Theorem 3.2 of [1] was flawed: Since $(F_\omega^R)_*(m_0|\Delta_{\sigma^{-n}\omega,0}, n \leq 1)$ is not a probability measure on $\Delta_{\omega,0}$ in general, the claim three lines below (3.9) that the sequence $\phi_{n,\omega}$ is [1] bounded, uniformly in n, is unfounded. (Indeed, even if F_ω^R is supposed piecewise affine, counter-examples may be constructed.)

Weixiao Shen kindly provided the following argument below to fix this proof.

We do not claim anymore that $\{h_\omega^{-1}\} \in \mathcal{F}_\beta^{\mathcal{K}_\omega}$ if (3.2) holds. See below how to deduce Lemma 5.1 from mixing of F and upper bounds from h_ω, without using lower bounds for h_ω, on which the proof of (5.1) depended. The lower bound for h_ω is not used elsewhere.

Corrected proof of Thm 3.2:

Instead of working with F_ω^R, we directly work with F_ω (just like in the proof of Sublemma 5.5 (1)). More precisely, for each ω and $n \geq 0$, let μ_n^ω be the push-forward of $m_0|\Delta_{\sigma^{-n}\omega,0}$ by $F^n_{\sigma^{-n}\omega}$. This push-forward is a probability measure on the tower Δ_ω, absolutely continuous with respect to m. Estimate (3.9) implies that the densities φ_n^ω of the μ_n^ω belong to $\mathcal{F}_\beta^+$, with constants $\sup_n C_{\varphi_{n,\omega}} < \infty$.

Recall that (3.3) says that $m(\Delta_\omega) < \infty$ for almost every ω.

In the application to unimodal maps, (3.3) means that we may view almost every Δ_ω as a compact interval. Thus, Arzela-Ascoli gives for almost every ω a subsequence $n_\ell \to \infty$ so that $\frac{1}{n_\ell}\sum_{k=0}^{n_\ell-1}\varphi_k^\omega$ converges to the density of a probability measure on Δ_ω.

In the general case, (3.3) implies that for almost all ω, there is a subsequence $n_\ell \to \infty$ so that $\frac{1}{n_\ell}\sum_{k=0}^{n_\ell-1}\mu_k^\omega$ converges in the weak-(*) topology to a probability measure on Δ_ω, absolutely continuous with respect to m.

In both cases, by the diagonal principle, for almost every ω, we can find a

[1]Note also the – unimportant — typo in the def. of $\phi_{n,\omega}$ where $\frac{1}{n}$ should read $\frac{d}{dm_0}\frac{1}{n}$.

sequence n_m such that for each integer N

$$\frac{1}{n_m}\sum_{k=N}^{n_m-1}\mu_{k-N}^{\sigma^{-N}\omega}$$

converges to a probability measure $\mu_{\sigma^{-N}\omega}$ on the tower $\Delta_{\sigma^{-N}\omega}$, absolutely continuous with respect to m. By construction, $(F_\omega)_*(\mu_\omega)=\mu_{\sigma\omega}$ for almost every ω. This gives the claimed absolutely continuous sample sationary probability measure.

Next, the construction implies that the density h_ω of μ_ω is bounded uniformly for almost all ω. In particular, $\{h_\omega, \text{ a.a. } \omega\}$ belongs to $\mathcal{F}_\beta^+\cap\mathcal{F}_\beta^{\mathcal{K}_\omega}$, where $\mathcal{K}_\omega$ is bounded uniformly over almost all ω.

This ends the proof of Theorem 3.2, with the statement on h_ω^{-1} removed. (The proof does not require (3.2).)

For the sake of comparison with the original proof of Theorem 3.2, we note that the restrictions $\hat\nu_\omega$ of μ_ω to $\Delta_{\omega,0}$ give a family of finite measures, which is invariant under F_ω^R, in the sense that for almost all ω and each $E\subset\Delta_{\omega,0}$

$$\hat\nu_\omega(E)=\sum_{\ell=1}^{\infty}\hat\nu_{\sigma^{-\ell}\omega}((F^R)^{-1}(E)\cap\Delta_{\sigma^{-\ell}\omega,0})\,.$$

(A priori $\hat\nu_{\sigma^{-n}\omega}(\Delta_{\sigma^{-n}(\omega),0})=\mu_{\sigma^{-n}(\omega)}(\Delta_{\sigma^{-n}(\omega),0})$ may depend on ω and n, i.e., our assumptions do not guarantee a common normalisation factor.)

Note also that h_ω may vanish at (x,ℓ) for $\ell\geq 1$, but then it vanishes identically on the element of $\mathcal{Z}_\omega$ containing (x,ℓ).

Corrected proof of Lemma 5.1:

First note that, instead of (5.1), we may use the weaker claim $\int_\Omega V_\omega^\ell dP>0$ to show that $\int\exp[-\upsilon V_{\sigma^{\tau_{i-1}}}^{\tau_i-\tau_{i-1}}]\,dP(\omega)<1$ if $\upsilon>0$ is small enough. We explain how to deduce $\int_\Omega V_\omega^\ell dP>0$ from mixing of F:

Let [2] $\hat V_\omega^\ell=\mu_\omega(\Delta_{\omega,0}\cap F_\omega^{-n}(\Delta_{\sigma^n\omega,0}))$. Then a simple application of the mixing property of F implies

$$\lim_{\ell\to\infty}\int_\Omega \hat V_\omega^\ell dP(\omega)=\mu_\epsilon(\Lambda)^2\,.$$

It is easy to prove that $\mu_\epsilon(\Lambda)>0$. So we obtain a positive lower bound for $\int_\Omega\hat V_\omega^\ell dP(\omega)$, for ℓ large. Since the density h_ω of μ_ω is bounded from above, $V_\omega^\ell>C\hat V_\omega^\ell$, so $\int V_\omega^\ell dP$ is bounded away from zero. This ends the proof.

References

[1] V. Baladi, M. Benedicks, V. Maume-Deschamps, Almost sure rates of mixing for i.i.d. unimodal maps, Ann. E.N.S., 35, 77–126 (2002).

[2] $F_\omega^{-\ell}(\Delta_{\omega,0})$ should be replaced by $F_\omega^{-\ell}(\Delta_{\sigma^\ell\omega,0})$ in the definition of V_ω^ℓ on page 92 of the paper.

Correcting the proof of Corollary 5.2 in *Almost sure rates of mixing for i.i.d. unimodal maps* (V. Baladi, M. Benedicks, V. Maume-Deschamps) Ann. E.N.S. (2002)

July 1, 2017

We thank Wael Bahsoun, Christopher Bose, and Marks Ruziboev, who pointed out to us that the proof of Corollary 5.2 in [1, p. 94] was flawed: In the inequality in the 9th line of this proof, the supremum over ω of the sum must be replaced by the sum of the suprema over ω. The next inequality then does not hold. We explain below how to recover Corollary 5.2 *up to replacing the set M'_q on p. 93 by the smaller set*

$$M'_q = \{(\omega, x, x') \in \bigcup_{\omega \in \Omega} (\{\omega\} \times \Delta_\omega \times \Delta_\omega) \,|\omega \in M_q^{\{\tau_i^\omega(x,x'), 0 \le i \le q\}} \text{ and } \tau_q^\omega(x, x') > [q^{1/v}]\},$$

where the constant $v \in (0, 1/4)$ is determined by the estimate (I) in the proof of Proposition 5.6. Since Corollary 5.2 is only used to obtain the estimate (II) in the proof of Proposition 5.6, and since $q \le n^v < q + 1$ there (for the same v), the rest of the paper is not affected.

Corrected proof of Corollary 5.2: First note that Corollary 5.2 is not used to prove Lemma 5.3, Lemma 5.4, Sublemma 5.5, or the bound for (I) in Proposition 5.6. We can thus postpone the statement and proof of Corollary 5.2 to just before the proof of Proposition 5.6.

Next, for fixed ω, q, and $\{\tau_i, i = 1, \dots, q\}$ with $\tau_j - \tau_{j-1} \ge \ell_0$, set

$$(\Delta^2_\omega)^{\{\tau_i\}} = (\Delta_\omega \times \Delta_\omega) \cap \{\tau_i^\omega(x, x') = \tau_i, i = 1, ...q\}.$$

By adapting the argument used to bound (I) in the proof of Proposition 5.6 on pp. 97-98, using n_5 defined there, we can prove that for any ω, any $n > n_5(\omega)$, and any fixed $\tau_0, \tau_1, \dots, \tau_q$ so that $\tau_{q-1} \le n < \tau_q$, we have (set $\tau_i - \tau_{i-1} = k_i$)

$$m \times m((\Delta^2_\omega)^{\{\tau_i\}}) < C(\epsilon) e^{-n^1/4}.$$

Summing over the k_i so that $\sum_{i=1}^{q} k_i \le n$ like on p. 98, we still find $C(\epsilon)e^{-n^{1/4}}$, up to changing the constant. (This step is the key estimate.)

Since we can assume that $\mathcal{K}_\omega^2 < \kappa^{-q/8}$ (because $P(\mathcal{K}_\omega^2 > \kappa^{-q/8})$ is small by (3.8)), it suffices to bound the sum of

$$\kappa^q \kappa^{-q/8} \sum_{n > \tilde{v}^8 |\log \kappa|^8} \sum_{\tau_i : \tau_{q-1} \le n < \tau_q} \sup_{\omega \in \Omega^{\{\tau_i\}}, n_5(\omega) < \tilde{v}^8 |\log \kappa|^8} m \times m((\Delta_\omega^2)^{\{\tau_i\}})$$

(for which we can use the key estimate) with

$$\kappa^q \kappa^{-q/8} \sum_{n < \tilde{v}^8 |\log \kappa|^8} \sum_{\tau_i : \tau_{q-1} \le n < \tau_q} \sup_{\omega \in \Omega^{\{\tau_i\}}} m \times m((\Delta_\omega^2)^{\{\tau_i\}})$$

(which is tractable by hand) and $P(n_5(\omega) > \tilde{v}^8 |\log \kappa|^8)$ (which is $\le \kappa^{\tilde{v}}$ by p. 97). We conclude by choosing $\tilde{v} > 0$ suitably.

References

[1] V. Baladi, M. Benedicks, V. Maume-Deschamps, Almost sure rates of mixing for i.i.d. unimodal maps, Ann. E.N.S., 35, 77–126 (2002).