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Spectra of hyp. ana. maps and flows.
Corr fcts, Fredholm dets and ζ-fcts (1)

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Overview :

* Real-analytic setup.

I - 1D expanding map f

(*) | Koopman oper. $B \to B \circ f$
| Natural meas (Birkhoff) Corr fcts
| Perron-Frob. oper. on $A_\infty(D)$

(*) | Repr as Cauchy integrals
| Fredholm dets / Residues.

(*) | Return to Koopman
| Dual space represent. $A_\infty(\hat{\mathbb{C}} \setminus \bar{D})$
| Multiplic. oper. repr.
| Thm on spectra of ~~hyp~~ 1D-exp ana maps.

II - 2D hyp maps (analytic) Axiom A attractors.

(*) Koopman
Natural / SRB meas. Corr fcts.

Thm on spectra and Fredh dets for 2D-hyp ana maps.

(*) Representation of the P-F oper on a fct space.

Fredholm dets

(*) Thermodyn Formalism proof of Thm

III - 3D hyp ana flows
Axiom A attractor

(*) Thm on spectra and ζ-fct.

(*) suspension of a R-a map

(*) Corr. fcts of flow
Fourier transf. ~>

"Modified"
corr-fct and determinant

Appendix { Fredholm 1905 (Riesz-Nagy '56)

log-Tr formula A matrix $N \times N$

$\log \det(1-\lambda A) =$

$\sum \log(1-\lambda\lambda_i) =$

$-\sum \frac{\lambda^n}{n}\sum \lambda_i^n = -\sum \frac{\lambda^n}{n} \mathrm{Tr}\, A^n$

$\det(1-\lambda A) = \exp\left(-\sum \frac{\lambda^n}{n}\mathrm{Tr}\, A^n\right)$

Integral oper : $k \in C^0([a,b]^2)$

$K\varphi(s) = \int_a^b k(s,t)\varphi(t)\,dt$

approx : N int's, a ... b

$\det(1-\lambda K_N) =$

$\det\begin{pmatrix} 1-\lambda k_{11}\Delta t & -\lambda k_{12}\Delta t & \cdots \\ -\lambda k_{21}\Delta t & 1-\lambda k_{22}\Delta t & \cdots \\ \vdots & & \end{pmatrix}$

$= 1 - \lambda \sum k_{ii}\Delta t + \frac{\lambda^2}{2!}\sum\sum \det\begin{vmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj}\end{vmatrix} \Delta t^2$

$\mp \cdots$

$\to 1 - \lambda \int k(t,t)\,dt + \frac{\lambda^2}{2}\iint \det\begin{vmatrix} k(s,s) & k(s,t) \\ k(t,s) & k(t,t)\end{vmatrix} ds\,dt$

$\mp \cdots$

$= \sum d_k \lambda^k, \quad d_0 = 1$

Hadamard $\Rightarrow$

$\det |(a_1)\cdots(a_n)| \leq \|a_1\|_{\ell^2}\cdots\|a_n\|_{\ell^2}$

$|d_m| \leq \frac{1}{m!}\left(\sqrt{m}\,\|k\|_\infty\right)^m (b-a)^m$

$\leq m^{-m/2} \times$ exponential

↳ tends to zero faster than any exp.

Grothendieck '56

$K = \sum \lambda_i e_i \otimes l_i$
ham i

$\sum |\lambda_i|^{2/3} < \infty \Rightarrow$

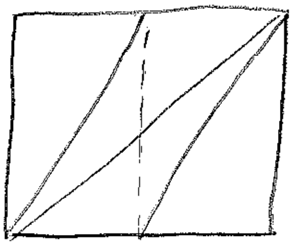
$\mathrm{Tr}\, K = \sum \lambda_i (l_i, e_i)$ is a genuine trace

$\det(1-\lambda K) = \cdots$ holds

1D expanding map.

Perron-Frob operator

- ex: $f(x) = 2x + \frac{k}{2\pi}\sin(2\pi x) \mod \mathbb{Z}$
$-1 < k < 1$

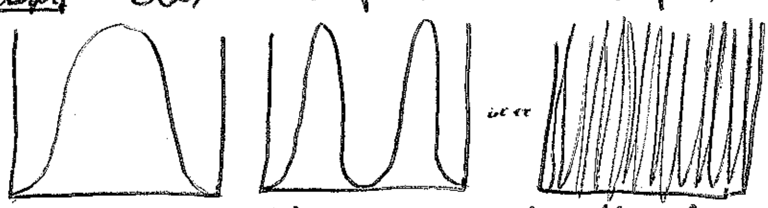


real-analytic expanding map on $S^1 = \mathbb{R}/\mathbb{Z}$
⇝ on $I = [0,1]$. (discontinuous).

$\exists!$ "natural" measure μ_{nat} (erg+mix)
For Leb. a-e. $x_0 \in [0,1]$, $B \in C^0([0,1])$

$$\frac{1}{N}\sum_0^{N-1} B \circ f^n(x_0) \longrightarrow \int_0^1 B \, d\mu_{nat}$$

$B(x)$ $\quad B \circ f(x)$ ..... $B \circ f^k(x)$



analyticity "deterioration" of the Koopman op $B \to B \circ f$.

Instead:

Def (P-F-oper.)

$$\int_I B \circ f \cdot A \cdot dx \stackrel{def}{=} \int_I B \cdot \mathcal{L}A \, dy$$

$$\mathcal{L}A(y) = \sum_{x: fx = y} \frac{1}{(\pm) f'(x)} A(x)$$

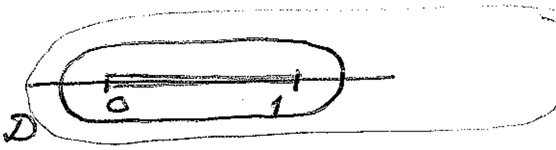
composition with contracting inverses ⇝ analyt. improving.

Thm (Ruelle-P-F) $0 < \alpha < 1$

$\mathcal{L} \in L(C^\alpha([0,1]))$ has a "spectral gap". $\exists\, h \in C^\alpha$, $\nu \in (C^\alpha)'$ (in fact, $\nu \in (C^0)'$ so is a measure) with
$\mathcal{L}h = h$ $\quad \nu\mathcal{L} = \nu$ $\quad \rho_{sp}(\mathcal{L} - h \otimes \nu) < 1$.
$\nu(h) = 1$
⇝ $d\mu_{nat} = h \cdot dx$
here ν = Leb. meas.

Analytic set up.

$D \subset \mathbb{C}$ domain nghd of I. (regular)



$\hat{f}_0 : D \to \mathbb{C}$ $\quad \hat{f}(z) = f(z)$
$\hat{f}_1 : D \to \mathbb{C}$ $\quad \hat{f}(z) = f(z) - 1$ (no mod $\mathbb{Z}$)
$\hat{f}_i(D) \supset\supset D$ $\quad f_i$ univalent
$\hat{f}_i(\partial D) \cap D = \emptyset$.

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

$X = A(D) \stackrel{def}{=} C^\omega(D) \cap C^0(\bar{D})$
sup-norm.

$$\mathcal{L}A(w) = \sum_i (\pm) \oint_{\partial D} \frac{A(z)}{f_i(z) - w} \frac{dz}{2\pi i} = \sum \oint \psi_i'(w) \frac{A(z)\,dz}{z - \psi_i(w) 2\pi i}$$

Integral operators with a smooth kernel.

Fredholm dets: (1905)

$K\phi(t) = \int k(t,s)\phi(s)\,ds$
comp. int J $\quad$ cont on $J \times J$

$$d(\lambda) = \det(1 - \lambda K) = \exp\left(-\sum \frac{\lambda^n}{n} \operatorname{tr} K^n\right)$$

extends to an entire fct of λ.

$d(\lambda) = 0 \Leftrightarrow 1/\lambda \in sp(K)$

order of zero = multipl. of e.val.

$\operatorname{tr} K = \int_J k(t,t)\,dt$

$\operatorname{tr} K^2 = \iint k(t,s)\,k(s,t)\,ds\,dt$

etc...

Calculus of residue

(3)

$$\mathrm{tr}\,\mathcal{L} = \sum_i \oint \frac{1}{\tilde f_i(z) - z}\frac{dz}{2\pi i}$$

$$= \sum_i \frac{1}{\tilde f'(z_i^*) - 1} \;/\; z_i^* = \tilde f_i(z_i^*)$$

$$= \sum_{x: f(x)=x} \frac{1}{f'(x)-1}$$

More generally

$$\mathrm{tr}\,\mathcal{L}^n = \sum_{x\in \mathrm{Fix} f^n} \frac{1}{|Df^{(n)}(x)-1|}$$

$$d(z) = \exp\left(-\sum \frac{z^n}{n}\sum_{x\in \mathrm{Fix} f^n}\frac{1}{|Df^{(n)}(x)-1|}\right) \text{ entire in } z$$

(but be careful: Don't use $f\circ f\circ\ldots\circ f$ in the Cauchy int^s. instead use pre-images and symb dyn)

Returning to Koopman:

Dual space of $X = A_\infty(D)$

$X' \underset{\text{isom}}{\cong} A_\infty(\hat{\mathbb{C}}\setminus\bar D)$ {analyt in $\hat{\mathbb{C}}\setminus \bar D$, vanishing at ∞.

$l\in A_\infty(\hat{\mathbb{C}}\setminus\bar D)$, $A\in A_\infty(D)$

$$\langle l, A\rangle \equiv \oint_{\partial D} l(z)A(z)\frac{dz}{2\pi i}$$

- ex: Lebesgue meas on $I=[0,1]$:

$$\int_0^1 A(x)dx = \oint_{\partial D}\left(\int_0^1 \frac{1}{z-x}dx\right)A(z)\frac{dz}{2\pi i}$$

cut along $[0,1]$

$$leb_{[0,1]}(z) = \int_0^1 \frac{1}{z-x}dx = \log\frac{z+1}{z}$$

representation in $A_\infty(\hat{\mathbb{C}}\setminus\bar D)$

- exer2: s.t. $leb_{[0,1]}\mathcal{L} = leb_{[0,1]}$ in $A_\infty(\hat{\mathbb{C}}\setminus\bar D)$

- exer1: $\oint \frac{l(\omega)}{f(z)-\omega}\frac{d\omega}{2\pi i} \notin A_\infty(\hat{\mathbb{C}}\setminus\bar D)$ unless $f(z)=az+b$.

instead:

$$\mathcal{L}_i' l(z) = \oint \frac{l(\omega)}{z-\psi_i(\omega)}\psi_i'(\omega)\frac{d\omega}{2\pi i}$$

$\langle \mathcal{L}_i' l, A\rangle = \langle l, \mathcal{L}_i A\rangle$

$l\in A_\infty(\hat{\mathbb{C}}\setminus\bar D)$ $\quad A\in A_\infty(D)$

$$\mathcal{L}_i' l(z) = \oint_{\partial D}\frac{1}{z-\xi}\oint_{\partial D}\frac{l(\omega)}{f(\xi)-\omega}\frac{d\omega\, d\xi}{2\pi i\cdot 2\pi i}$$

$z\notin \bar D$

$$= \oint \frac{l(\omega)}{z-\psi_i(\omega)}\psi_i'(\omega)\frac{d\omega}{2\pi i}$$

push integrals $\Rightarrow$ $\mathcal{L}_i' : A_\infty(D)' \to A_\infty(\hat{\mathbb{C}}\setminus\bar D)$ $(\subset A_\infty(D)')$

also an integral operator (same kernel as for $\mathcal{L}_i$, transposed vars $\Rightarrow$ same trace and det).

Multiplication oper:

$A\in A_\infty(D)$ Ban. alg.

$$M_A\phi(z) \equiv A(z)\phi(z)$$

$$M_A' l(\omega) = \oint_{\partial D}\frac{1}{z-\omega}A(\omega)l(\omega)\frac{d\omega}{2\pi i}$$

$\langle M_A' l, \phi\rangle = \langle l, A\phi\rangle$

(caution: slightly sing kernel)

Prop:

$$\mathrm{tr}\,\mathcal{L}^n M_A = \sum_{x\in \mathrm{fix} f^n}\frac{A(x)}{|Df^{(n)}(x)-1|}$$

$$\lim_{n\to\infty} \mathrm{tr}\,\mathcal{L}^n M_A = \int A\, d\mu_{nat}$$

$\parallel$

$\nu(M_A h)$

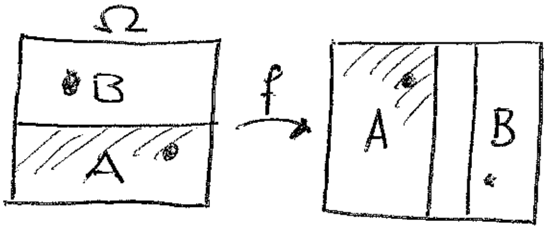
Then poles of $\hat C(\omega)$ $\subset$ zeroset $d(e^{i\omega})$

$$C(k) \stackrel{\text{def}}{=} \int B\circ f^k A\, d\mu_{nat} = \nu(M_B\mathcal{L}^k M_A h)$$

$$\text{FT: } \sum C(k)e^{ik\omega} = \nu(M_B(1-e^{i\omega}\mathcal{L})^{-1}M_A h)$$

II 2D hyperbolic map

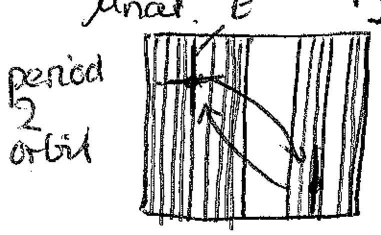
- ex:



Axiom A attract. Λ

SRB. measure / attractor

supp on $\Lambda = \bigcap_{k\geq 0} f^k \Omega$ = Cantorset × [0,1]



preserved under (small) R-a perturb^s.

but foliation in general only $C^{1+\alpha}$ (or C^α in higher dim)

Thermodyn Formalism (SRB) looses R-a but shows:

$$\int A \, d\mu_{nat} = \lim_n \sum_{x \in \text{fix} f^n} \frac{A(x)}{Df^{(n)}_{|E^u}(x)}$$

↑ only Hölder

$$C_{BA}(k) = \int B \circ f^k A \, d\mu_{nat}$$

$$\hat C_{BA}(\omega) = \sum e^{ik\omega} C_{BA}(k)$$

a priori merom in a strip around the real axis.

Thm A, B analytic on Λ (extends to)

2) $\hat C_{BA}(\omega)$ merom in the cmplx plane

3) poles of $\hat C(\omega)$ $\Leftrightarrow$ {recipr. zero of a Fredholm det $\Leftrightarrow$ {evals of $\mathcal{L}$ resp. + sp gap

following

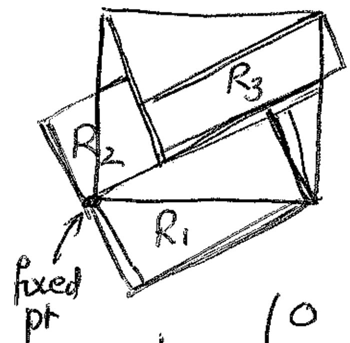
1) The F-det is entire:

$$d(z) = \exp\left(-\sum_{n\geq 1} \frac{z^n}{n} \operatorname{tr} \mathcal{L}^n\right)$$

$d(1)=0$ simple zero

$$\operatorname{tr} \mathcal{L}^n = \sum_{x \in \text{Fix} f^n} \frac{1}{|\det(Df^n - \mathbb{1})|}$$

Anosov.



$$fx = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Transition matrix

$$\underline{\underline{t}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Markov map, but count incorrectly fixed pts:

$\operatorname{tr} t = 0 \neq \# \text{Fix} f = 1$

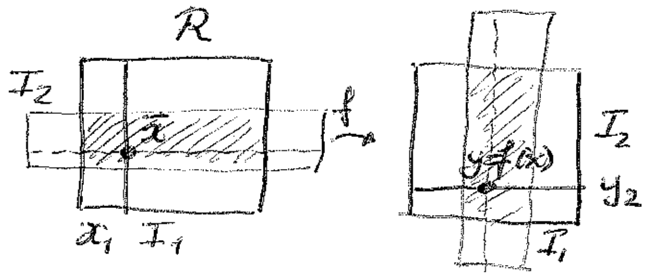
$\hat f_{\underline{\underline{t}}} : \{R_1 \cup R_2 \cup R_3\} \to \{R_1 \cup R_2 \cup R_3\}$

SRB for $\hat f_{\underline{\underline{t}}}$ project to SRB for f.

$$\int A \, d\mu_{nat} = \int A \circ \pi \, d\hat\mu_{nat} = \lim_n \sum_{x \in \text{fix} \hat f^n} \frac{A \circ \pi(x)}{D\hat f^{(n)}_{|E^u}(x)}$$

Hyperbolic analytic map between rectangles.

- modulo symbolic dyn. only one such rectangle.



Model ex: $f(x) = \left(\frac{x_1}{2}, 2x_2\right) + \dots$

$I_1 = I_2 = [-1, 1]$.

Notation $f(x) = (f_1(x_1, x_2), f_2(x_1, x_2))$

Want to calculate:

$$\iint_{R \cap f^{-1}R} B \circ f(x)\, A(x)\, dx_1 dx_2 =: C_{BA}(1)$$

Idea: Change of coordinates, introducing "pinning coord^s"

$$R \cap f^{-1}R = \{ (x_1, \varphi_s(x_1, y_2)) : x_1 \in I_1, y_2 \in I_2 \}$$

image

$$fR \cap R = \{ (\phi_u(x_1, y_2), y_2)) : x_1 \in I_1, y_2 \in I_2 \}$$

Model ex: $\phi_s(x_1, y_2) = \frac{y_2}{2} + \dots$

$\phi_u(x_1, y_2) = \frac{x_1}{2} + \dots$

"so pinning coord^s are contractions! good news

$$C_{BA}(1) = \iint_{I_1 I_2} B(\varphi_u(x_1, y_2), y_2)\, A(x_1, \varphi_s(x_1, y_2))\, \partial_2 \phi_2\, dy_2 dx_1$$

"unstable" derivative

But pinning coord^s of iterated map not iterates of pinning coord^s! bad news.

Def: $f : R \rightrightarrows R$ is a hyperbolic analytic map between rectangles iff $\exists$ complex nghds (smooth bdry) $I_1 \subset D_1 \subset \mathbb{C}$ $I_2 \subset D_2 \subset \mathbb{C}$ and analytic extensions of pinning coor^s so that

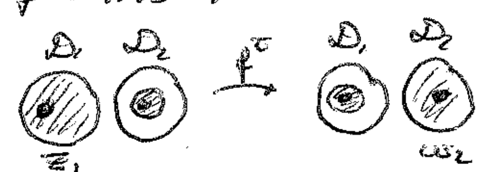
$$\phi_s : \bar{D}_1 \times \bar{D}_2 \to D_2$$
$$\phi_u : \bar{D}_1 \times \bar{D}_2 \to D_1$$

$$f(z_1, \phi_s(z_1, \omega_2)) = (\varphi_u(z_1, \omega_2), \omega_2)$$
$$\forall z_1, \omega_2 \in D_1 \times D_2$$

Prop: $\exists!$ pinning coor^s for all iterates of f

$\phi_s^\tau : \bar{D}_1 \times \bar{D}_2 \to D_2$ $\phi_u^\tau : \bar{D}_1 \times \bar{D}_2 \to D_1$

$f^\tau(z_1, \phi_s^\tau(z_1, \omega_2)) = (\phi_u^\tau(z_1, \omega_2), \omega_2)$



(orbit at times $0 .. \tau$ stays in $\bar{D}_1 \times \bar{D}_2$)

Function space and repres of a "PFO"

$X = A_\infty((\hat{\mathbb{C}} \setminus \bar{D}_1) \times D_2)$

Dual space (contains:)

$X' = A_\infty(D_1 \times (\hat{\mathbb{C}} \setminus D_2))$

Green's fct.

$$G^{(\tau)}_{\underline{\omega}, \underline{z}} = \frac{1}{\omega_1 - \phi_s^\tau(z_1, \omega_2)} \, \frac{\partial_2 \phi_s^\tau(z_1, \omega_2)}{z_2 - \phi_s^\tau(z_1, \omega_2)}$$

$\eta(\underline{z}) \quad z_1 \in \hat{\mathbb{C}} \setminus \bar{D}_1, \; z_2 \in D_2$ given

$$\overset{(\tau)}{L}\eta(\underline{\omega}) \equiv \oint\oint_{\partial D_2 \partial D_1} G^\tau_{\omega, z} \, \eta(z) \, \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i}$$

analytic in a ngh of $\omega_1, \omega_2 \in (\hat{\mathbb{C}} \setminus D_1) \times \bar{D}_2$

~~asymptotical~~

$\boxed{\text{Prop: } \mathcal{L}^{(\sigma)} \mathcal{L}^{(\tau)} = \mathcal{L}^{(\tau)} \mathcal{L}^{(\sigma)} = \mathcal{L}^{(\sigma + \tau)} \quad \text{(2-dim res calculus)}}$

Nuclear operator with trace

$$Tr\, \mathcal{L}^{(\tau)} = (\pm) \oint\oint \frac{d\omega_1}{2\pi i} \frac{d\omega_2}{2\pi i} \frac{1}{(\omega_1 - f_1(\omega_1, \omega_2))} \frac{1}{(f_2(\omega_1, \omega_2) - 1)}$$

$$= \frac{1}{|\det(Df - \mathbb{1})|} \quad \text{(2-dim res. calculus)}$$

> **Thm** $\hat{f}_{ji} : R_i \to R_j \quad (t_{ji} = 1)$ collection of hyp ana maps. Then
>
> $d(z) = \exp(-\sum \frac{z^n}{n} Tr \mathcal{L}^n)$
>
> $Tr\, \hat{\mathcal{L}}^n = \sum_{x \in fix \hat{f}^n} \frac{1}{|\det(Df^n - 1)|}$
>
> extends to an entire fct. in the complex plane.

$\hat{\mathcal{L}}$ acts as a matrix oper. $(\hat{\mathcal{L}}_{ji} : X_i \to X_j)_{t_{ji} = 1}$ on the product ΠX_i

$d(\lambda) = 0 \iff 1/\lambda \in sp\, \hat{\mathcal{L}}$
order multipl.

Multiplication operator representing $A \in A_\infty(D_1 \times D_2)$

$$(M_A \eta)(\omega_1, \omega_2) = \oint_{\partial D_1} \frac{A(\xi_1, \omega_2)}{\omega_1 - \xi_1} \eta(\xi_1, \omega_2) \frac{d\xi_1}{2\pi i}$$

(multiplication on the bdry of D_1 and projected back into X)

on 2-dim res calc:

$$\oint\oint_{\partial D \partial D} \frac{1}{\lambda_{11} z_1 + \lambda_{12} z_2} \, \frac{1}{\lambda_{21} z_1 + \lambda_{22} z_2} \, \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i}$$

$|\lambda_{11}| > |\lambda_{12}| \quad |\lambda_{21}| < |\lambda_{22}|$

$$= \frac{1}{\lambda_{11}} \oint \frac{1}{-\frac{\lambda_{21}\lambda_{12}}{\lambda_{11}} z_2 + \lambda_{22} z_2} \frac{dz_2}{2\pi i}$$

$$= \frac{1}{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}} = \frac{1}{\det \Lambda}$$

Alternatively α holom 2-form of max degree

$$\int_{\partial D \times \partial D} \alpha_\Lambda = \int_{\Lambda(\partial D \times \partial D)} \frac{1}{\det \Lambda} \alpha_{\mathbb{1}}$$

$$= \int_{\Lambda_t(\partial D \times \partial D)} \frac{1}{\det \Lambda} \alpha_{\mathbb{1}} = \int_{\partial D \times \partial D} \alpha_{\mathbb{1}} \frac{1}{\det \Lambda}$$

$$\int_\Lambda A\, d\mu_{SRB} = \lim_n \sum_{x\in \mathrm{Fix} f^n} \frac{A(x)}{Df^n(x)|_{E^u}}$$ conv at expon rate for $A \in \mathcal{O}^\omega(\Lambda)$

$$\frac{1}{\det(Df^n(x)-1)} = \frac{1}{(\Lambda_u - 1)(1-\lambda_s)}$$

$$\left(Df^n(x) = \begin{pmatrix} \Lambda_u & 0 \\ 0 & \lambda_s \end{pmatrix}\right) \quad \geq C\theta^{-n}, \quad \leq C\theta^n$$

$$= \frac{1}{\Lambda_u}\left(1-\frac{1}{\Lambda_u}\right)^{-1}(1-\lambda_s)^{-1}$$

$$= \frac{1}{\Lambda_u}\left(1+\mathcal{O}(\theta^n)\right)$$

$$\int A\, d\mu_{SRB} = \sum_{\mathrm{Fix} f^{n}} \frac{A(x)}{Df^n|_{E^u}(x)} + \mathcal{O}(\theta^n)$$

$$= \sum \frac{A(x)}{|\det(Df^n(x) - 1\!\!1)|} + \mathcal{O}(\theta^n)$$

$$= \mathrm{Tr}(\mathcal{L}^n M_A) + \mathcal{O}(\theta^n)$$

$A \equiv 1 \Rightarrow \quad 1 = \mathrm{Tr}\,\mathcal{L}^n + \mathcal{O}(\theta^n)$

Fredholm det theory $\Rightarrow$ 1 simple eval
$P = h \otimes \nu$ proj.
+ sp-gap.

$\mathcal{L}$ trace class
$|\mathrm{Tr}(\mathcal{L} Q)| \leq \mathrm{Const}\, \|Q\|, \ \forall Q \in L(X)$

$\mathrm{Tr}(\mathcal{L}^n M_A) =$

$\mathrm{Tr}(P M_A) + \underline{\mathrm{Tr}(\mathcal{L}(1-P)\mathcal{L}^{n-1} M_A)}$

$\quad \nu(M_A h) \qquad \leq \mathrm{const}\, \theta^{n-1} \|M_A\|$

$$\int A\, d\mu_{SRB} = \nu(M_A h) + \cancel{\mathcal{O}(\theta^n)}$$
$$= \lim \sum_{x\in \mathrm{Fix} f^n} \frac{A(x)}{|\det(Df^n(x) - 1\!\!1)|}$$

$$C_{BA}(t) = \int B\circ f^t A\, d\mu_{nat}$$

$$= \lim_n \sum_{x\in \mathrm{Fix} f^{n+t}} \frac{B\circ f^t(x) A(x)}{|Df^{(n+t)}(x) - 1\!\!1|}$$

$$= \mathrm{Tr}(\hat{\mathcal{L}}^n M_B \hat{\mathcal{L}}^t M_A)$$

$$\to \nu(M_B \hat{\mathcal{L}}^t M_A h)$$

$$\hat{C}(\omega) = \sum_{t\geq 0} C_{BA}(t) e^{it\omega}$$

$$= \nu(M_B (1 - e^{i\omega}\hat{\mathcal{L}})^{-1} M_A h)$$

$\{$poles of $\hat{C}(\omega)\}$ discrete set

$\subset \{\omega : 1 \in \mathrm{sp}\, e^{i\omega}\hat{\mathcal{L}}\}$

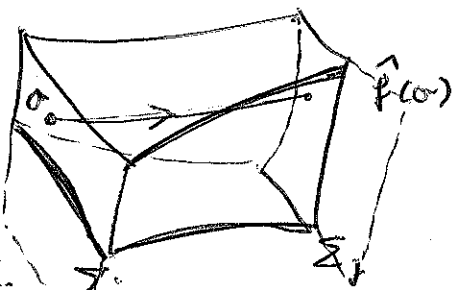
$\equiv \{\omega : \hat{d}(e^{i\omega}) = 0\}$

Fredholm det

The flow case.

$U \subset \mathcal{M}$; $\phi^t U \subset\subset U$, $t > 0$; $\Lambda = \bigcap \phi^t U$

Axiom A attractor.



analytic "almost" markov

transversal section

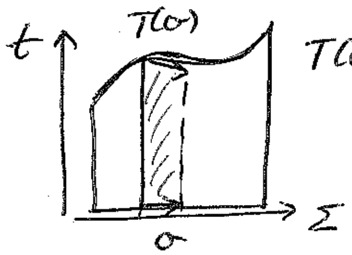
transition from $\Sigma_i \to \Sigma_j$

$$\hat{f}_{ji}(\sigma) = \phi^{T_{ji}(\sigma)}(\sigma)$$

hyperbolic analytic map.

$\mu_{nat}^{\hat{f}}$ SRB-meas on $\{\Sigma_i\} \circlearrowleft \hat{f}$

μ_{nat}^{ϕ} SRB meas on $\Lambda \circlearrowleft \phi^t$



Then $d\mu_{nat}^{\phi} \cong d\mu_{nat}^{\hat{f}} \otimes dt$ on (σ, t); $\sigma \in \Sigma$, $0 \leq t \leq T(\sigma)$.

identif: $(\sigma, T(\sigma)) \sim (\hat{f}(\sigma), 0)$

$(\sigma, T(\sigma)+t) \sim (\hat{f}(\sigma), t)$

Jacobian $\begin{pmatrix} D\hat{f} & D_\sigma T \\ 0 & 1 \end{pmatrix}$

Given analytic obs B, A

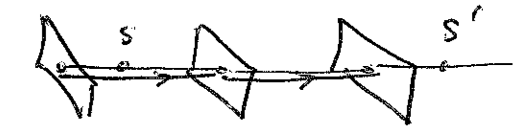
$\omega \in \mathbb{R}$

$$\tilde{B}_\omega(\sigma) = \int_0^{T(\sigma)} e^{i\omega s'} B \circ \phi^{s'}(\sigma)\, ds'$$

$$\tilde{A}_\omega(\sigma) = \int_0^{T(\sigma)} e^{-i\omega s} A \circ \phi^{s}(\sigma)\, ds$$

$$\int A\, d\mu_{nat}^{\phi} = \int \tilde{A}_0(\sigma)\, d\mu_{nat}^{\hat{f}}$$

$$= \nu(M_{\tilde{A}_0} h)$$



Correlation fct

$$C_{BA}(\tau) = \int_\Lambda B \circ \phi^\tau A\, d\mu_{nat}^{\phi}$$

Fourier transf $\operatorname{Im}\omega > 0$

$$\hat{C}(\omega) = \int_\Lambda \int_{\mathbb{R}_+} B \circ \phi^t e^{i\omega t} A\, d\mu_{nat}^{\phi} dt$$

$x \in \Lambda$, $x = \phi^s(\sigma)$

$\sigma \in \Sigma$, $0 \leq s \leq T(\sigma)$.

+ holom(ω) (first transition ... $k=0$ different ($s' > s$))

$$\hat{C}(\omega) = \sum_{k \geq 0} \iiint_\Sigma B \circ \phi^{s'} \circ \hat{f}^k (\sigma)$$

$$\exp\left(i\omega\left(\sum_0^{k-1} T \circ \hat{f}^j(\sigma) + i\omega(s'-s)\right)\right)$$

$$A(\phi^s(\sigma))\, d\mu_{nat}^{\hat{f}}(\sigma)\, ds\, ds'$$

$$= \sum_k \int \tilde{B}_\omega \circ \hat{f}^k \cdot e^{i\omega \sum T \circ \hat{f}^j} \tilde{A}_\omega\, d\mu_{nat}^{\hat{f}}$$

$$= \sum_k \nu(M_{\tilde{B}_\omega} (M_{\exp(i\omega T)} \hat{\mathcal{L}})^k \tilde{A}_\omega h)$$

$$= \nu(M_{\tilde{B}_\omega} (1 - M_{\exp(i\omega T)} \hat{\mathcal{L}})^{-1} \tilde{A}_\omega h)$$

poles must belong to ω for which $1 \in sp(M_{\exp(i\omega T)} \hat{\mathcal{L}})$ or

$$\det(1 - M_{\exp(i\omega T)} \hat{\mathcal{L}}) = 0$$

Note that $sp(\hat{\mathcal{L}})$ has no physical meaning